Life of a scientist dedicated to mathematics

Founder of the School of Theory of Functions in Azerbaijan, prominent scientist in the field of function theory and approximation theory, Ibrahimov Ibrahim Ibish oglu, was born on February 28, 1912 in Gargabazar village of Fizuli region of Azerbaijan. His father, Ibish, died in 1917, and he was raised by his mother Mansuma widowed at the age of 30, and his uncle Yusif. He received his first education in 1921 at the age of 9 at the school of Mullah Yusif in the village of Pirahmadli, one kilometer from his village. A few months later, he continued his education at a five-grade primary school opened in his village. After graduating from that school in 1925, he entered the newly opened Peasant Youth School in Fizuli in 1926 and continued his education there until May 1928.

From September 1928 he continued his education at the Ganja Agricultural College. In 1931, he was transferred to the preparatory course of the Azerbaijan State Pedagogical Institute under the Ganja Pedagogical College. Ibrahim, who successfully completed the preparatory course, was admitted to the Azerbaijan State Pedagogical Institute in 1931. In October 1934, Ibrahim Ibrahimov was called up for military service. He served in the army until May 1935, continuing his higher education at the Pedagogical Institute. In May of that year, he was released from military service.

While studying at the Pedagogical Institute, he was taught Mathematical Analysis by Bahram Yusifzadeh and Mammadbey Efendiyev, Analytical Geometry by Eynulhayat khanum Yusifzadeh, Physics by Rahimbey Malikov and from the theory of real variable functions by Ashraf Huseynov. Ibrahim Ibrahimov graduated from the institute with honors in 1935 and was appointed a mathematics teacher at a secondary school in Zangilan district, according to the appointment of the Ministry of Education. After working at the school for several months, with the help of his friends, he was appointed a mathematics teacher at the Oil Refinery College in the Montin settlement of Baku (now Narimanov district). In early 1936, he continues his education in a training course led by professors of the Azerbaijan State University A.S. Kovanko, A.A. Bukhshtab and Yaroslav Borisovich Lopatinsky. Those who did not have a university education at that time had to take a preparatory course to be admitted to graduate school.

Ibrahim Ibrahimov, who successfully completed the postgraduate course in November 1936, was admitted to the university's postgraduate course in the theory of functions. AA Buchshtab, who gave lectures on higher algebra, number theory and the theory of complex variables at the university, was appointed his scientific adviser. In order to increase his knowledge of the Russian language during his postgraduate studies, young Ibrahim of began to translate the two-volume book "Differential and Integral Calculus" of Konstantin Alexandrovich Posse and Ivan Ivanovich Privalov into Azerbaijani. At that time, there were no textbooks and teaching aids in the Azerbaijani language on the subjects taught at the Faculty of Physics and Mathematics. The work of teachers working in the Azerbaijani department was very difficult. Within a few months, he completed the translation of the first volume and published it. In 1935-1936, he also worked as a lecturer at the Institute of Advanced training of engineers and an assistant at the Azerbaijan State University.

Ibrahim Ibrahimov left for Moscow in December 1937 to continue his postgraduate studies. He was received by the director of the Steklov Institute of Mathematics, academician Andrei Nikolayevich Kolmogorov, who checked his documents and scheduled an exam two days later. After the exam, Alexander Osipovich Gelfond, one of the most influential mathematicians of the time, agreed to become his supervisor.

Ibrahim Ibrahimov, who met with his supervisor once a week, listened to his lectures and actively participated in the seminars he led, prepared his dissertation in late April 1939 and submitted it to his supervisor for a year and a half. On May 20, 1939, he defended his dissertation "On the completeness of the system of some analytical functions" at Moscow State University under the guidance of A.O.Gelfond. His official opponents were well-known scientists of the time I.I. Privalov and Mikhail Alekseyevich Lavrentyev. Thus, on May 20, 1939, Ibrahim Ibrahimov became the first Azerbaijani to receive the degree of Candidate of Physical and Mathematical Sciences (Doctor of Philosophy in Mathematics).

After the defense, he returned to Baku and began lecturing at the Azerbaijan State University, the Azerbaijan Pedagogical Institute and the Promacademy of the Ministry of Heavy Industry on higher algebra, the theory of functions of complex variables, and mathematical physics equations. In 1939-

1947 he worked as the head of the "Theory of Functions" department of the Azerbaijan State University. In 1940, Completes and publishes the translation of K. Rosse and I.I.Privalov's second book "Integral Calculus".

In 1944, Ibrahim Ibrahimov was sent to Moscow to work on his doctoral dissertation. He writes in his memoirs that as soon as he arrived in Moscow, he met with A.O. Gelfond and stated that he had come to Moscow for two years to work on his doctoral dissertation. He was informed about my scientific activity, the results I received and so on. After listening to me carefully, he said that Academician Sergei Natanovich Bernstein would soon hold a seminar at the Faculty of Mechanics and Mathematics of Moscow State University. He is looking for talented, promising students, try to attend his seminar. In addition, it would be good if you take part in the seminar I conduct together with M.V.Keldysh and A.I.Markushevich. Don't hurry, call me as soon as you get a new scientific result, you will report at the seminar after getting acquainted with the text of your report and giving my consent.

In 1945, he speaks 7 times at the seminar led by S.N.Bernstein. One of these reports was about "finding the best approximation of a function with a real singular point by means of polynomials" put forward by Bernstein. By the way, it should be noted that this seminar was attended by well-known scientists of the time A.O.Gelfond, V.L.Goncharov, N.I.Ahiezer, S.M.Nikolsky, S.B.Stechkin, N.K.Bari, D.E.Menshov and others.

In the spring of 1945, during a break in Bernstein's seminar, Ibrahim met Keldish at his home and presented him a new scientific paper on the completeness of analytical functions. After looking carefully at the work, Keldish suggests him to cansider the following two problems:

1. To compare the results with the results of Gelfond and Markushevich on this issue;

2. Build an example that shows that it is impossible to improve the obtained result.

Keldysh advises that the work can be prepared for publication only after solving these problems. After completing Keldysh's assignments and discussing the results with him again, in Keldysh's opinion he publishes it in one of the most influential journals of the period Doklady Acad. of Sci. of the USSR.

During six months in Moscow, he presented 3 scientific works to the journal "News of the USSR Academy of Sciences" and 2 scientific works to the journal "Reports of the USSR Academy of Sciences" with the presentations of Keldysh and Bernstein.

In the summer of 1945, when the scientific seminars ceased, he worked on the problems posed by Bernstein in the real domain, and on the problems proposed by Keldysh in the complex domain. He met with Bernstein in September and showed him the results in both directions. Bernstein got acquainted with the work, asked a few questions and made a proposal. Ibrahim notes in his memoirs that taking into account Bernstein's questions and suggestions played a major role in preparing these works for publication.

In October 1945, Ibrahim Ibrahimov presented to Keldish the first results related to Carleman's generalized problem. He recommends him to report on these results at the seminar. After the seminar, he meets with Keldish and takes into account his advice, and soon solves the so-called "generalized problem of Carleman."

In November 1945, he presented his work "Polynomial Approximation of a Function with a singular Point" to Gelfond. After reviewing the manuscript, he says that the results obtained here are very close to solving the so-called "Bernstein hypothesis". Talk about these results at Bernstein's seminar, and think about how to solve the problem when a singular point is on the appoximation segment. He talks about his results at the seminar and Bernstein likes the report. He advises to continue working in this direction. After working on this problem for several months, Ibrahim presented his findings at Bernstein seminar in February 1946, proving that the "Bernstein hypothesis" is true.

At the end of 1945, Ibrahim continued his research with Keldysh, proving that it was impossible to improve his conclusions on the completeness of the above-mentioned system of analytical functions, and published it with Keldysh in Mathematical Sbornik, one of the most influential journals of the period.

Ibrahim Ibrahimov reports on the unpublished results at the seminar "Theory of Functions of Complex Variables" conducted by Keldish, Gelfond and Markushevich in the second half of January 1946.

In March 1946, he completed his research on the convergence of the Abel-Goncharov interpolation series" and published it in the Mathematicheckiy Sbornik with Keldysh's opinion.

In May 1946, in the seminar Ibrahim Ibrahimov reported on his scientific results on "The process of Lagrange trigonometric interpolation". After the lecture, Keldysh says to Gelfond "I think it's time for him to defend his doctoral dissertation." Gelfond also supports this idea.

Thus, Ibrahim Ibrahimov is invited to combine and systematize his scientific results on complex variable functions and submit them to the Scientific Council of the Faculty of Mechanics and Mathematics of Moscow State University as a doctoral dissertation.

A year and a half after his arrival in Moscow, he prepared his doctoral dissertation and submitted it to the Defense Council.

At that time, scientists who had worked part of their doctoral dissertations in the regions were admitted to the doctoral program for two years and sent to the center. Ibrahim writes in his memoirs that my completion of my doctoral dissertation for a year and a half caused a sensation in Moscow. Some people could not hide their surprise. I reassured myself, thinking that there are many who throw stones at fruit trees.

On June 14, 1946, Ibrahim Ibrahimov defended his doctoral dissertation at the Scientific Council of the Faculty of Mechanics and Mathematics of Moscow State University. 40 out of 60 members took part in the defense. Enough is provided. The defense was very quiet and no one spoke out against it. Official opponents Keldysh, Gelfond and Goncharov expressed positive opinions about the dissertation. However, for some reason, after the defense, the counting commission said that 31 people took part in the voting, which was not enough. As a result, the Scientific Council decided to declare the defense unsuccessful due to insufficiency and postpone it to the autumn of 1946. Thus, his defense was postponed and Ibrahim has to return to Baku.

In his memoirs, Ibrahim writes, "From December 1944 to June 1946, during my year and a half in Moscow, I published 12 large scientific articles rich in new scientific results in the most prestigious journals of the USSR. Despite the fact that such eminent scientists as Gelfond, Goncharov and Nikolsky spoke positively about my scientific results, i was not allowed to defend my doctoral dissertation without giving any good reasons.

Undoubtedly, this incident affected him very badly, and for six months he could not concentrate and do anything. He started working as a senior researcher at the Institute of Physics and Mathematics at ASU. He also teaches at both the university and API. In 1946-1947 he prepared and published four new scientific articles "On the best polynomial approximation of functions of real variables". In May 1947, he was elected the head of the Department of Mathematical Analysis at API, where he worked until 1958.

In the summer of 1947, Ibrahim traveled to Moscow for a few days to present his new scientific findings to Bernstein, Keldysh and Gelfond. They advise to prepare a new dissertation in the real domain and submit it to the Scientific Council of the Steklov Institute of Mathematics of the USSR Academy of Sciences. At that time, the director of the institute was academician I.M.Vinogradov, Keldysh and I.G.Petrovsky were his deputies.

Ibrahim Ibrahimov came to Moscow in December 1947 to defend his doctoral dissertation. Prepares the dissertation and abstract and submits it to the Defense Council of the Steklov Institute of Mathematics. The council approves Bernstein, Nikolsky and Markushevich as opponents. The defense is scheduled for January 12. The chairman of the Defense Council was academician Vinogradov. In addition to opponents, Keldysh and Gelfond also speak at the defence. Thus, on February 12, 1947, Ibrahim Ibrahimov is defending his doctoral dissertation on "Some problems of the theory of interpolation and approximation of functions" at the Steklov Institute of Mathematics in Moscow.

Upon his return from Moscow, he organized a research seminar for teachers and graduate students at the department headed by him at the Azerbaijan State Pedagogical Institute. This seminar was attended by teachers and graduate students (doctoral students) of the Department of Mathematical Analysis and Theory of Functions of ASU, API and Correspondence Pedagogical Institute. The weekly seminar had more than 40 participants.

In May 1947, Ibrahimov began to teach a special course on "Number Theory" in the third year at ASU, and "Polynomial approximation of functions of real variables" in the fourth year. In 1948, he went to Ganja and gave lectures on mathematical analysis at the Pedagogical Institute named after Hasan bey Zardabi.

In January 1952, Ibrahim Ibrahimov in Moscow at the international conference dedicated to the 50th anniversary of the birth of academician M.A.Lavrentyev reported on the subject "Approximation of functions of complex variables in infinite domains by entire functions", and in 1955 he made a report in the III Congress of the USSR matematicians held in Moscow State University on the topic "Entire functions of finite degree".

He started working at the Institute of Physics and Mathematics of the Academy of Sciences of the Azerbaijan SSR in 1958. In the summer of 1958, he was invited to Sofia, the capital of Bulgaria, where he gave lectures for a month.

In June 1959 he was included in the organizing committee of the All-Union Conference in Leningrad (now St. Petersburg), chaired by Academician V.I.Smirnov, and delivered two reports. In 1958-1959 he worked as the head of the department of the theory of functions at the Institute of Physics and Mathematics of the Azerbaijan Academy of Sciences. In October 1959, two institutes: the Institute of Physics, the Institute of Mathematics and Mechanics were established on the basis of this institute. Ibrahim Ibrahimov was appointed director of the Institute of Mathematics and Mechanics in 1959 and held this position until 1963. At the end of the same year, he was relieved of his post as director of the institute by his own application, and from 1963 to the end of his life he headed the department of the theory of functions of the institute.

Ibrahim Ibrahimov was elected a corresponding member of the Academy of Sciences of the Azerbaijan SSR in December 1959 at the age of 47.

In June 1960, in the IV All Union Congress of mathetaticians held in Leningrad University he delivered a speech on "Some inegualities in the class of entire functions of finite degree", and in august 1962 in the international congress of mathematicians on "Inegualities between the Lebesgue generalized norms of entire functions of finite degree", and he delivered a speech at the Second All-Union Conference on Constructive Theory of Functions held in Baku from October 8 to 14 in 1962 on "Some extreme problems for linear operators in the class of finite-degree complete functions."

In 1965, at the invitation of the University of Chernivtsi, Ukraine, he gave lectures to 4th and 5th year students of the Faculty of Mathematics on "Approximation Theory", "Interpolation Theory of Complex Variable Functions" and "Some Problems in the Theory of Analytical Functions".

In 1965, Ibrahim Ibrahimov's research on "Extreme properties of finite degree entire functions", "Best quadratic and cubic formulas" and "Approximation of functions of complex variables by complete functions" is reflected in five of his works published in the central press.

In August 1966, he spoke at the International Congress of Mathematicians in Moscow on "Extreme Properties of Analytical Functions in the Semi-Plane", at which he chaired one of the sections. This congress was attended by 20 mathematicians from Baku.

In January 1968, at the age of 56, he was elected a full member of the Academy of Sciences of the Azerbaijan SSR. In 1969 he was invited to an international conference in Budapest, where he delivered a one-hour report on "Bernstein's work in the field of constructive theory of functions."

In August 1970, he delivered a speech at the International Congress in Nice, France, on "The Completeness of the System of Some Analytical Functions." Ashraf Huseynov and Mirabbas Gasimov from Azerbaijan also took part in the congress.

In 1971, the Moscow publishing house Nauka published a monograph entitled "Methods of interpolation of functions and some of their applications." At the All-Union Conference in Kharkov in early 1971, he made a report on "The study of the problem of the uniqueness of entire functions by interpolation methods."

In 1972, Ibrahim Ibrahimov was awarded the Honorary Decree of the Supreme Soviet of the Azerbaijan SSR on the occasion of his 60th birthday and an article about him written by Nikolsky, Markushevich and Yevgrafov was published in one of the authoritative journals of that time "Uspekhi Matematicheskikh Nauk".

Here it is worth recalling what academician Azad Mirzajanzadeh wrote about him. He writes: To imagine its greatness, it is enough to mention the Keldysh-Ibrahimov theorem and the article of academician Bernstein on the contributions of Ibrahim Ibrahimov to the theory of approximation, published in the journal "Uspekhi Matematicheskikh Nauk". In June 1972, he presented a paper on "A method for studying the completeness of the system of analytical functions" at an international congress held in Varna, Bulgaria.

In October 1973, he was invited by the Czechoslovak Academy of Sciences and the University of Prague to give lectures on the theory of approximation and interpolation of functions. In March and April 1974, he delivered a series of lectures at universities in Prague, Brno and Bratislava.

In 1974, he published 8 scientific works on the completeness, multiplicity and basicity of analytical functions. In 1975, at the international conference held in Bulgaria he gave a report on "Basicity conditions of some systems of analytical functions in the circle", at the international conference in Kaluga in 1976, "Classic constructive characteristics of complex variable functions", in August 1976 in Ufa "On multiple completeness of the system of analytical functions in the Keldysh sense".

In 1977 Ibrahim Ibrahimov's 8 works "On the constructive characteristics of classes of functions of complex variables", "Multiple completeness of classes of different analytical functions", "The best mean approximation of analytical functions", "Study of the convergence of singular integrals in a complex domain" and some related issues were published.

Ibrahim Ibrahimov was elected President of the Azerbaijan Mathematical Society in January 1977.

He published 3 scientific works in 1978 and 6 in 1979. At the end of 1979, his third monograph, "The Theory of Approximation of Entire Functions", was published.

In 1980 he spoke at an international conference in Ufa on "The direct and inverse problems of the theory of aproximation in a class of functions."

Ibrahim Ibrahimov notes in his memoirs that for more than 40 years, starting in 1946, regardless of the circumstances, the scientific seminar organized by me on the theory of functions continued its work on a regular basis. First of all, the scientific results of our employees, graduate students, dissertations, dissertations of more than 100 scientists from other cities were discussed here. I think that the scientific seminar on mathematics plays the role of laboratories in chemistry and physics.

The academician is the author of more than 170 scientific works, 3 monographs, 5 textbooks, translated 2 textbooks from Russian into Azerbaijani. 53 of his articles were published in the Dokl. of the USSR, one of the most influential scientific journals of the time, and 9 in the journal Izv AN USSR, 3 in Matem.sb., 3 in Uspekhi Matem. Nauk, 2 in Matem. Zametki.

Under his leadership, more than 40 candidates of sciences were prepared, and later 6 students became doctors of sciences. Ibrahim Muallim always approached his students with high demands, along with increasing their scientific potential, made great efforts to form them as a person, and became a worthy example to young people with his scientific and pedagogical activities.

In 1982, Ibrahim Ibrahimov turned 70 years old, on the occasion of which a large article about him by Nikolsky, Markushovich, Kudryavtsev and Yevgrafov was published in the journal Uspekhi Matem. Nauk.

On February 26, 1982, the 70th anniversary of Ibrahim Ibrahimov was celebrated in the Great Hall of the Academy of Sciences of the Azerbaijan SSR.

Academician Ibrahim Ibrahimov's work has always been highly valued by the state, he was awarded the Order of Friendship of Peoples in 1986, and on April 16, 1986 he was awarded the honorary title of "Honored Scientist".

On March 16, 1992, the 80th anniversary of academician Ibrahim Ibrahimov was celebrated in the Great Hall of the Academy.

Academician Ibrahim oglu Ibrahimov has passed away on October 6, 1994 and was buried in the first Alley of Honors.

Books

1. Fundamentals of the Theory of Odds, Baku, Azerneshr, 1955, page 380
2. Fundamentals of Beer Theory, Baku, Azerneshr, 1957, page 498
3. Mathematical Analysis Course, Textbook, Baku Azerneshr, 1962
4. Extreme properties of finite functions of a finite function, Baku, Azerbaijan Publishing House. USSR, 1962, 316 Art.
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6. Theory of approximation by whole functions, Baku, Izd. USSR, 1979, 468 Art.
7. Studies on modern problems of constructive theory of functions, Baku, Izd. USSR, 1965, 165 Art.
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9. Special Issues in Function Theory, Baku, Azerbaijan Publishing House. USSR, 1980, 184 Art.
10. A Life Dedicated to Mathematics, Baku, Elm Publishing House, 2002, 328 pages.

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1. Posse K, Privalov I, Differential Accounting Course, Baku, Azerneshr, 1944, page 413.
2. Posse K, Privalov I, Integrated Accounting Course, Baku, Azerneshr, 1952, page 543.

Undergraduate students Candidates of Science

1. Asghar Akhundov, 1950,
2. Hidayat Abdullayev, 1951,
3. Hussein Husseinov, 1952,
4. Ali Safarov, 1953,
5. Bahram Yusifzadeh, 1953,
6. Ali Hasanov, 1954,
7. A. Kartashyan, 1954,
8. Qazanfar Cafarli, 1955,

9. Arif Ismailov, 1955,
10. Rashid Mammadov, 1955,
11. Munavvar Efendieva, 1955,
12. Gümrah Keferli 1956,
13. Yahya Mammadov, 1956,
14. Mohammad Mammadov, 1957,
15. Kardashkhan A. Orucov, 1958,
16. Karim Khalafov, 1958,
17. Nusrat Sadykhov, 1962
18. We found Akhmedov, 1963,
19. Malik-Bakhish Babayev, 1963,
20. Abusalat Khamidov, 1963,
21. Akif Haciyev, 1964,
22. Camal Mammadkhanov, 1964
23. Nariman Sabziyev, 1964,
24. Rafiq Aliyev, 1965,
25. Farhad Nasibov,
26. Charkaz Agamaliyev, 1967
27. Zulfukar Hasanov, 1967
28. Abakar Mukhtarov, 1970,
29. Kardashkhan Y. Orucov, 1970,
30. Waqif Shahverdiyev, 1973
31. Elman Kasimov, 1974
32. Nizami Shikhaliyev, 1977
33. S.S. Lynchuk, 1978,
34. L.F. Kushnirchuk, 1978,
35. Yusuf Batchaev, 1980,
36. S.R. Orucov, 1981,
37. Ashot Nersesyan, 1981,
38. Ilham Aliyev, 1983,
39. Rasim Ihadov, 1982,
40. V.M. Muradov, 1982,
41. Khanlar Rustamov, 1983,

Doctors of Science

1. Rashid Mammadov, 1962, 2. Akif Khaciyev, 1981 3. Camal Mammadkhanov, 1981. 4. Nudae Naqnibida 1986

Ibrahim Ibrahimov, who has close scientific relations with prominent mathematicians of the time Mstislav Vsevolodovich Keldish, Sergey Natanovich Bernstein, Sergey Mikhailovich Nikolsky, AO Gelfond, MA Lavrentyev and others in the field of constructive theory of complex and real variable functions, is rightly considered a representative of the Moscow School of Mathematics. Among his researches on various problems of the theory of complex and real variable functions, the solution of the Carleman problem occupies an important place. Together with academician Keldysh, he found the exact sign for the convergence of Newton's interpolation series in all classes of complete functions, and together with Gelfond he solved the problem of "two points" using the methods of interpolation theory. I.Ibrahimov's works on "Convergence of Abel-Goncharov interpolation series, Lagrange trigonometric interpolation series and integration series of rational functions" belong to the second direction of his research.

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Motion of the gas-liquid mixture in the system of the reservoir-pipeline when connecting to the main line of the new source

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The heat exchange process can have a significant effect on the dynamics of the gas-liquid mixture in the system of the reservoir-pipeline. The rheological properties of the gas-liquid mixture are sensitive to changing the temperature of its surrounding medium. When the gas-liquid mixture moves through the pipeline, heat exchange occurs between the flow of the gas-liquid mixture and the environment surrounding it. This leads to a change in the viscosity and density of the gas-liquid mixture and as a result of this to the change in the dynamics of its motion. In addition, the density of the gas-liquid mixture is even highly dependent on the pressure, which is also reflected on the dynamics of its movement.

This work constructed a model, taking into account the heat exchange process between the flow of a gas-liquid mixture in the lifting pipe and its surrounding medium and the solutions of boundary value problems are given. In the first approximation, the effect of changes in the rheological properties of the gas-liquid mixture is determined depending on the temperature on the dynamics of its motion. Analytical formulas obtained, allowing to determine the pressure dynamics on the bottom of the well and the productivity of the layer depending on the parameters of the system. Numerical calculations were carried out in practical values of the system parameters.

Diffusion transport of in-situ generated gas in fluid-saturated porous media

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In situ generated CO_2 gas affects the change in the viscosity ratio of fluids involved in the reaction, the oil volume coefficient and residual oil saturation. There is a change in the viscosity of reservoir fluids depending on the gas dissolved in it, i.e., the concentration of $\mathrm{CO_2}$ [1].

The relevance of studying the processes of generated gas transfer in fluids is due to a number of applied problems. Another problem concerns the transfer of generated gas under the influence of a temperature wave in porous media saturated with formation fluids.

It is known that part of the formed $\mathrm{CO_2}$ dissolves in water and oil (up to saturation), while another part forms immobile gas bubbles held by surface tension forces. Formation of a temperature wave propagating in the porous medium significantly changes the solubility profile, affecting the diffusion transport of the saturated solution. This leads either to "locking" of a part of the released CO_2 in the bubble layers or, on the contrary, to the enhanced diffusive "transport".

Similar processes in porous media occur with natural gas bubbles. Low temperatures allow saturation of saturating fluids with gas, while increasing the temperature leads to the transition of excess gas from the solution into the gas phase and the formation of bubbles. Thus, the thermal wave can not only saturate the water and oil with generated CO_2, but also leads to "pumping" of this gas into the bubble layers.

In this work we investigated the process of gas generation and the effect of temperature waves on transport in fluid-saturated porous media.

The paper presents analytical expressions for the time-average diffusion flow of slightly soluble substances through a liquid filling a porous half-space, provided that the solution is saturated everywhere and the temperature fluctuates. The regularity of gas solubility in reservoir water was formulated, which is directly proportional to reservoir pressure at a constant temperature, and depends on the mineralization of the liquid.

Systems with complicating factors such as thermal diffusion or convective transfer are considered. A kinetic model for the transport of dissolved gas is being developed, which combines the equation of hydrodynamic dispersion with the diffusion transfer of dissolved gas between the saturated solution and the bubble phase.

The harmonic law of temperature change during gas formation is assumed. This is justified, firstly, by the fact that real profiles differ slightly from harmonic ones and, secondly, by a decrease in the depth of penetration into the array of higher harmonics. Then the temperature field will be determined by the thermal conductivity equation for a half-space with a specified boundary condition on the surface of the array and the requirement of temperature limitation at infinity [2, 3]:

$$\frac{\partial T}{\partial t} = \chi \frac{\partial^2 T}{\partial x^2}$$

$$x = 0 : T = T_0 + \theta_0 \cos(\omega t)$$

$$x \to \infty : T < \infty$$

where is the T_0 - average temperature, θ_0 - the amplitude of temperature fluctuations, χ - the thermal conductivity of the array, $\frac{2\pi}{\sigma}$ - the time period, the coordinate is calculated from the formation of gas in the direction of displacement. Thus, the following temperature distribution takes place in the array:

$$T = T_0 + \theta_0 e^{-kz} \cos(\omega t - kz), \quad k \equiv \sqrt{\frac{\omega}{2\chi}}$$

As a characteristic value of thermal conductivity, we take the thermal conductivity of water and oil. The wave number of the temperature wave equal to $k \approx 0,8$ m^{-1}. The pressure field is reservoir $P = P_r$.

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Study of the flow of heterogeneous fluids with variable component content and external environment

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Heterogeneous mixtures are used in many technological processes. In the flow of some classes of such mixtures in pipes and porous media, anomalous effects can be seen. For all these processes, the presence of the interface and dispersion between phases is considered one of the determining rheology characteristics.

Even when the concentration difference in the disperse systems changes, there is an increase in the stability of deposition, spontaneous formation (structuring) of phase structures. The main issue is the kinetics of structural changes, as a result of which the hydrodynamic parameters of the system depend on external influences results in increase or decrease in fluid flow. The process of thixotropic spontaneous dissociation and restoration of binding in structural systems is represented by many authors in the form of a kinetic equation [1-2]:

$$-\frac{dN_t}{dt} = k_1 N_+^n - k_2 N_-^m, \tag{1}$$

where N_+ and N_+ - are concentrations of dissociated and non-dissociated bonds;

$k_{1,2}$ - coefficients of the rate of destruction and restoration of links;

n and m - coefficients determining the degree of dissociation and bond restoration by analogy with a chemical reaction; t – time.

Some aspects of the behavior of heterogeneous liquids under conditions of variable component composition of fluids and variable external conditions are considered in our work. Studies were carried out on a HAAKE Rheostress-600 viscometer at adjustable shear rates and a temperature control temperature equal to 25^{o}C. The research method consisted in placing the studied samples of oil and an oil polymer mixture between the coaxial cylinders of the rheo-viscosimeter, one of which is movable. At certain angular shear rates, the values of the shear stress τ(Pa) were determined and the effective viscosity η_{ef} (MPa·s) was estimated. The experimental results showed that the equilibrium state between the shear rate and the shear stress is established with some delay. Fig. 1 shows the characteristic rheological curves obtained as a result of laboratory studies for a sample of resin-asphaltene oil containing 7.0 vol. % of asphaltenes.

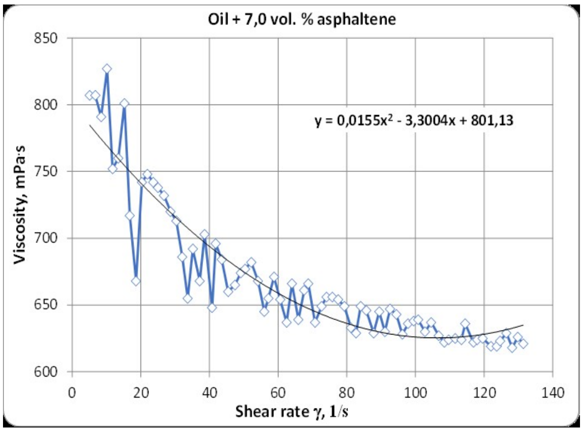

Fig. 1 Rheological characteristics of the oil sample with 7 vol. % asphaltene

This means that there are amplitudes of viscosity change at certain shear rates, which correspond to the transition from the deformation mode with small amplitudes to large ones. The process of spontaneous relaxation-thixotropic damped oscillation of the destruction-restoration of bonds in a structured system can be described as follows.

Let ω_0 be the natural frequency of the system – the attenuation coefficient. Then the attenuation process is represented by the differential equation [2 - 4]:

$$\frac{\partial^2 x}{\partial t^2} + 2\xi\omega_0 \frac{\partial x}{\partial t} + \omega^2 x = 0 \tag{2}$$

The characteristic roots of the equation are as

$$\begin{aligned} \lambda_1 &= \omega_0 \left(-\xi + \sqrt{\xi^2 - 1} \right) \\ \lambda_2 &= \omega_0 \left(-\xi - \sqrt{\xi^2 + 1} \right) \end{aligned} \tag{3}$$

In this case, weak attenuation occurs – weak attenuation corresponds to $\xi < 1$.Then the solution of the characteristic equation is two complex-conjugate roots:

$$\lambda_{1,2} = -\omega_0 \xi \pm i\omega_0 \sqrt{1 - \xi^2} \tag{4}$$

The solution of the equation can be written as follows:

$$x(t) = e^{-i\omega_0 t} \left(C_1 \cos(\omega_0 t) + C_2 \sin(\omega_0 t) \right)$$

where, $\omega_d = \omega_0 \sqrt{1-\xi^2}$- the natural frequency of damped oscillations, constants C_1 and C_2 are determined from the initial conditions.

$$x(0) = a, \ \frac{\partial x}{\partial t}(0) = b \tag{5}$$

The value τdetermined from $\xi = \frac{1}{\tau}$, is the relaxation time, and the amplitude decreases in time.

Taking into account the peculiarities of the heterogeneous fluid flow with varying component composition and variable external conditions is relevant in the problems of liquids filtration and transport. It is established that at certain volume fractions of the dispersed phase and the value of the shear rate, an improvement in the characteristics of the flow and mutual displacement of fluids is take place.

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On compactness of sets in the Weighted Besov spaces

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In this abstract, we presented a compactness of sets in the weighted Besov spaces. For this purpose, a weighted modulus of continuity in a weighted Lebesgue space is defined, which differs from the modulus of continuity in the usual Lebesgue space. By means of the considered modulus of continuity and we formulate a theorem on the precompactness of sets from a weighted Besov space. As application of the considered modulus of continuity we establish an approximation theorem for family of averaging operators in weighted Lebesgue spaces by modulus of continuity.

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Generalized equation of stationary oil inflow to a well under steady modes.

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As is known, with changes in pressure and temperature in the reservoir, the physical properties of oil and reservoir also change (viscosity, volume coefficient of oil and permeability of the reservoir, etc.). With an increase in depression at the bottom of the well, there is a change (increase or decrease) in the operating capacity of the reservoir and a violation of the linear filtration law.

In this regard, the paper examines the influence of all the above factors on the shape of the indicator curves. A method of their quantitative interpretation for specific conditions for the study of wells under established regimes is proposed [1]. In addition, an attempt is made to quantify the influence of each factor separately.

With a two-membered filtration law, without taking into account gas segregation, the equations of stationary oil inflow to the well can generally be represented as:

$$q_{oil} + F(P_{well}) \cdot q_{oil}^2 = A_0(\varphi_{res} - \varphi_{well}) \tag{1}$$

where,

$$\varphi_{res} - \varphi_{well} = \int_{P_{well}}^{P_{res}} f_0\left(P\right) dP \tag{2}$$

$$A_0 = \frac{2\pi k\left(P_{res}\right) h_{eff}}{\mu_{oil}\left(P_{res}\right) a\left(P_{res}\right) ln(r_{cont}/r_{well})};$$

$$F(P_{well}) = \frac{\rho_{atm} k(P_{well})}{2\pi h\left(P_{well}\right) \mu\left(P_{well}\right) \cdot l \cdot \ln(r_{cont}/r_{well})};$$

$$f_0\left(P\right) = h^*(P) \cdot f(P);\, f(P) = \frac{k^*(P)}{\mu_{oil}^*\left(P\right) a^*(P)};\, h^*\left(P\right) = \frac{h(P)}{h_{eff}};\, k^*\left(P\right) = \frac{k(P)}{k(P_{res})};$$

$$\mu^*_{oil}(P) = \frac{\mu_{oil}(P)}{\mu_{oil}(P_{res})}; a^*(P) = \frac{a(P)}{a(P_{res})};$$

Here: h_{eff} and $h(p)$ are the effective and operating reservoir thickness, respectively: $k(P_{res}), \mu(P_{res})$, and $a(P_{res})$, are the values of the corresponding parameters at reservoir pressure, i.e. at $P = P_{res}, l$ - coefficient of macro-roughness. The rest of the designations are generally accepted.

In inference (1), given that the expression of the dependence of the complex of parameters $f_0(P)$ and the change in the operating power of the reservoir on the pressure can be written with a polynomial of a higher degree [2, 3]. According to the obtained equations, after some transformations, it is possible theoretically investigate various forms of indicator curves and give their general classification.

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Bernstein-Markov-Nikolskii-type inequalities in weighted Bergman space

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Let $\mathbb{C}$ be a complex plane and $G \subset \mathbb{C}$ be a bounded Jordan region, $L := \partial G$; Let $\wp_n$ denotes the class of all algebraic polynomials $P_n(z)$ of degree at most $n \in \mathbb{N}$. Let $\{z_j\}_{j=1}^m$ be the fixed system of distinct points on the curve L. We consider the generalized Jacobi weight function $h(z)$ which is defined as follows:

$$h(z) := \prod_{j=1}^{l} |z - z_j|^{\gamma_j}, \ \gamma_j > -2, \ j = 1, 2, ..., l, \ \ z \in \mathbb{C}, \tag{1}$$

Let $0 < p \leq \infty$ and σ be the two-dimensional Lebesgue measure. For the Jordan region G, we introduce:

$$\|P_n\|_{A_p(h,G)} := \left(\iint_G h(z) \, |P_n(z)|^p \, d\sigma_z \right)^{1/p}, \ p > 0;$$

In this work, we study the following

$$\|P_n^{(m)}\|_X \leq \lambda_n(G, h, p) \|P_n\|_Y \tag{2}$$

Bernstein ($X = Y = A_\infty$)-type, Markov ($X = Y = A_p, p > 0$)- type and Nikolskii ($m = 0; X = A_q, Y = A_p$, $0 < p < q < \infty$) - type inequalities in Bergman space for all polynomials $P_n \in \wp_n$ and any $m = 0, 1, 2, ...$, for various regions in the complex plane, where $\lambda_n := \lambda_n(G, h, p, m) > 0$, $\lambda_n \to \infty$, $n \to \infty$, is a constant, depending on the geometrical properties of the region G and the weight function h in general.

Inequalities of type (2) have been studied by many mathematicians since the beginning of the 20th century (see, [1], [2], [3] and others).

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Best approximation-preserving operators over Hardy space

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Let $T_n := K_n *$, $n \in \mathbb{Z}_+$, be the linear Hadamard convolution operator acting over Hardy space H^q, $1 \le q \le \infty$. We call T_n a best approximation-preserving operator (BAP operator) if $T_n(e_n) = e_n$, where $e_n(z) := z^n$, and if $\|T_n(f)\|_q \le E_n(f)_q$ for all $f \in H^q$, where $E_n(f)_q$ is the best approximation by algebraic polynomials of degree a most $n-1$ in H^q space and $E_0(f)_q = \|f\|_q$.

The main question is: what conditions on K_n are necessary and sufficient for T_n to be a BAP operator?

The problem was solved in case $n = 0$ in [1]. In this talk, we give a solution of the problem in general case.

Theorem 1. *Let $n \in \mathbb{Z}_+$, K_n be a function holomorphic in $\mathbb{D}$, $K_n(z) = z^n + O(z^{n+1})$ as $z \to 0$ and let $T_n = K_n *$ be a Hadamard convolution operator over H^1. Then T_n is a BAP operator over H^∞ if and only if*

$$\begin{cases} K_n(z) = z^n + O(z^{2n+1}) \ \text{as } z \to 0, \\ \mathrm{Re}\dfrac{K_n(z)}{z^n} \ge \dfrac{1}{2} \ \text{for all } z \in \mathbb{D}. \end{cases} \tag{1}$$

Moreover, (1) implies that T_n is a BAP operator over H^q space for $q \ge 1$.

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Optimization of nonlocal hyperbolic control problem

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In the paper there has been considered a class of control problem for hyperbolic equation

$$z_{tx}\left(t,x\right)=f\left(t,x,z\left(t,x\right),z_{t}\left(t,x\right),z_{x}\left(t,x\right),z\left(\tau_{0},\mu_{0}\right),u\left(t,x\right)\right)$$

$$\left(t,x\right)\in T\times X=\left[t_{0,},t_{1}\right]\times\left[x_{0},x_{1}\right], \tag{1}$$

Under linear nonlocal boundary conditions [1]

$$\left(V_{2}z\right)\left(t\right)\equiv\sum_{j=1}^{m}\left\{\int_{x_{0}}^{\mu_{j}}z_{t}\left(t,\mu\right)\gamma_{2,1}\left(t,\mu\right)d\mu+ right.$$

$$+\int_{x_{0}}^{\mu_{j}}z\left(t,\mu\right)\gamma_{2,0}\left(t,\mu\right)d\mu\right\}=l_{2}\left(t\right),$$

$$t\in T\left(V_{1}z\right)\left(x\right)\equiv z_{x}\left(t_{0},x\right)=l_{1}\left(x\right),\quad x\in X \tag{2}$$

$$V_{0}z\equiv z\left(t_{0},x_{0}\right)=l_{0}.$$

Control $u\left(t,x\right)$ satisfies the condition

$$u\left(t,x\right)\in V\subset R^{r}. \tag{3}$$

And optimality criteria has been given minimizing functional

$$S\left(u\right)=\varphi\left(z\left(\tau_{1},\mu_{1}\right),\ldots,z\left(\tau_{m},\mu_{m}\right)\right). \tag{4}$$

For this nonlocal hyperbolic control problem (1)-(4) under some conditions on the data [2] there has been given nessessary condition of optimality in view of maximum principle.

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The problem of fracture mechanics for a composite strip in the case of a longitudinal shear crack

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In the paper there is considered the problem of brittle fracture mechanics for a composite strip in the case of alongitudinal shear crack. Solution of the problem has been found and stress intensity coefficients have been determined by the Werner-Hoff method.

Let two homogeneous isotropicelastic materials with elastic properties E_j, v_j be rigidly coupled along the planes $\theta = \pm\alpha \quad (0 < r < \infty)$, with the first material (E_1, v_1) accupying the region $|\theta| \geq \alpha$ and with the second material (E_2, v_2) the region $|\theta| \leq \alpha$.

Thus, the composed medium contains a crack $|\theta| = 0, \quad 0 \leq r \leq l$. The boundary conditions of this problem have the form

$$|\theta| = \pi, \ \tau_{\tau\theta} = 0, \quad u_\theta = 0 \tag{1}$$

$$\theta = \alpha, \quad [\sigma_\theta] = |\tau_{\tau\theta}|, \quad [u_\theta] = [u_\tau] = 0 \tag{2}$$

$$\theta = 0, \quad r > 1, \ \tau_{\tau\theta} = 0, \ u_\theta = 0. \tag{3}$$

Here, $\sigma(r) \ \ (r \in [0,1])$ is given positive function. It is assumed that $E_1 \neq 0, \ \ v_1 \neq 0$. At infinity, the stresses are zero and the offsets disappear. Also,

$$\theta = 0, \quad r \to l + 0, \quad \sigma_\theta = \frac{K_I}{\sqrt{2\pi (r - l)}} \tag{4}$$

Using some transformations [1] from (4), we arrive at the Wiener-Hopf functional equation

$\Phi^+(P) + F(P) = G_1(P) + \Phi^-(P)$, where $G_1(P)$ is defined by (2). The solution to this problem has the form

$$\Phi^+(P) = \frac{-P\varphi^+(P) \cdot G^+(P)}{K^+(P)}, \Phi^-(P) = \frac{PG^-(P) \cdot K^-(p)\,\varphi^-(P)}{2(P + \delta + 1)}.$$

$K(P)$ is given in [2].The stress intensity coefficient is determined as follows

$K_I = \sqrt{2}\gamma_1, \quad \gamma_1 = \frac{1}{2\pi i}\int_L \varphi(t)dt$ and $k_I = F(P)\,\frac{K^+(P)}{PG^+(P)}$.

The integration contour is selected in the usual way [2].

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On an inverse problem for a parabolic equation in a domain with moving boundaries

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This paper considers the inverse problem of determining the unknown coefficient on the right-hand side of a parabolic equation in a domain with moving boundaries. An additional condition for finding the unknown coefficient, which depends on the variable time, is given in integral form. A theorem on uniqueness and "conditional" stability of the solution is proved.

Let $\gamma_1(t), \gamma_2(t)$ $t \in [0,T]$, $0 < T = const$ be the given functions, (x,t) be an arbitrary point in the bounded domain $D = (\gamma_1(t), \gamma_2(t)) \times (0,T]$, $[a,b]$ be the projection of the domain D into the axis OX.

We consider the following inverse problem on determining a pair of functions $\{f(t), u(x,t)\}$:

$$u_t - u_{xx} = f(t)g(x) \quad (x,t) \in D, \tag{1}$$

$$u(x,0) = \varphi(x) \quad x \in [\gamma_1(0), \gamma_2(0)], \tag{2}$$

$$u(\gamma_1(t),t) = \psi_1(t), \quad u(\gamma_2(t),t) = \psi_2(t), \; t \in [0,T], \tag{3}$$

$$\int\limits_{\gamma_1(t)}^{\gamma_2(t)} u(x,t)dx = h(t), \quad t \in [0,T], \tag{4}$$

where $g(x), \varphi(x), \psi_1(t), \psi_2(t), h(t)$ are the given functions, $u_t = \frac{\partial u}{\partial t}$, $u_x = \frac{\partial u}{\partial x}$, $u_{xx} = \frac{\partial^2 u}{\partial x^2}$.

In the paper, a theorem on the uniqueness and stability of the solution of the problem (1)-(4)

On solvability of one boundary value problem for a second order elliptic differential-operator equation containing a complex parameter

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In separable Hilbert space H, we consider the following boundary value problem for a second order elliptic differential-operator equations:

$$L(\lambda)\,u := \lambda^2 u(x) - u''(x) + Au(x) = f(x), \quad x \in (0,1), \tag{1}$$

$$\begin{aligned} &L_1 u := u'(0) + (\lambda^2 + \alpha_0\lambda + \alpha_1)u(0) = f_1, \\ &L_2 u := u(1) = f_2, \end{aligned} \tag{2}$$

where λ is a complex parameter.

Theorem 1. *Let the following conditions be fulfilled:*
1) A is a linear, closed, densely defined operator in H and $\|R(\lambda, A)\|_{B(H)} \leq \leq c(1+|\lambda|)^{-1}$ for $|arg\lambda| \geq \pi - \varphi$, where $\varphi \in (0,\pi)$ is some number, $c > 0$ is some constant independent on λ;
2) α_0, α_1 are any complex numbers and $\alpha_0 \neq 0$.

Then the operator $\mathbb{L}(\lambda) : u \to \mathbb{L}(\lambda)u := (L(\lambda)u, L_1(\lambda), L_2 u)$ for sufficiently large λ from the angle $|\lambda| \leq \frac{\varphi}{2} < \frac{\pi}{2}$ is an isomorphism from $W_p^2((0,1); H(A), H)$ to $L_p((0,1); H) \dot{+} (H(A), H)_{\theta_1,p} \dot{+} (H(A), H)_{\theta_2,p}$ where $\theta_1 = \frac{1}{2} + \frac{1}{2p}$, $\theta_2 = \frac{1}{2p}, p \in (1,\infty)$, and for these λ the following estimation is valid for solving the problems (1), (2)

$$\begin{aligned} &|\lambda|^2 \, \|u\|_{L_p((0,1);H)} + \|u''\|_{L_p((0,1);H)} + \|Au\|_{L_p((0,1);H)} \leq \\ &\leq C\Big[|\lambda|^2 \|f\|_{L_p((0,1);H)} + \sum_{k=1}^{2} \Big(\|f_k\|_{(H(A);\,H)_{\theta_k,p}} + |\lambda|^{2(1-\theta_k)} \|f_k\|_H \Big) \Big]. \end{aligned} \tag{3}$$

On discrete Hardy type inequality in variable eeighted variable Lebesgue spaces

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In this abstract, we establish a two-weight boundedness criterion of discrete Hardy operator and its dual operator in the scale of discrete weighted variable Lebesgue spaces. Moreover, we study the problem of compactness of the discrete Hardy operator in discrete weighted variable Lebesgue spaces. We also study a similar problem for the dual operator of discrete Hardy operator.

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Approximation of the Hilbert transform in Lebesgue spaces

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Let $u \in L_p(R)$, $1 \leq p < \infty$. The function

$$(Hu)(t) = \frac{1}{\pi} \int_R \frac{u(\tau)}{t - \tau} d\tau, \ t \in R,$$

is called the Hilbert transform of the function u.

It is known that (see [1]), the Hilbert transform of the function $u \in L_p(R)$, $1 \leq p < \infty$ exist for almost all $t \in R$. In case $1 < p < \infty$ the Hilbert transform is bounded operator in the space $L_p(R)$ and satisfies the equality $H^2 = -I$.

In [2], it is given an approximation of the Hilbert transform of a function u, which is analytic in the strip $\{z \in C : |Im(z)| < \alpha\}$ and satisfies additional conditions, by operators of the form $\frac{1}{\pi} \sum_{k=-\infty}^{\infty} \frac{u(t+k\delta)}{-k}$, and it is shown that, as $\delta \to 0$ this sequence uniformly converges to the function $(Hu)(t)$. Note that we can represent these operators in the form

$$(H_\delta u)(t) = \frac{1}{\pi} \sum_{k=-\infty}^{\infty} \frac{u(t + (k + 1/2)\delta)}{-k - 1/2}$$

The article is devoted to the approximation of the Hilbert transform of the function $u \in L_p(R)$, $1 < p < \infty$ by operators H_δ, $\delta > 0$.

Theorem 1. *For any $\delta > 0$ the operators H_δ are bounded in the space $L_p(R)$, $1 < p < \infty$, and the following inequality holds:*

$$H_\delta^2 = -I.$$

Theorem 2. *For any $\delta > 0$ the sequence of operators $\{H_{\delta/n}\}$ strongly converges to the operator H in $L_p(R)$, $1 < p < \infty$, that is the following inequality holds for any $u \in L_p(R)$:*

$$\lim_{n\to\infty} \|H_{\delta/n}u - Hu\|_{L_p(R)} = 0.$$

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On some properties of a singular operator on terms of generalized oscillation

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Let us consider the singular integral operator

$$A_k f(x) = \lim_{\varepsilon \to +0} \int_{R^n} \{K_\varepsilon(x-y) -$$

$$- \left(\sum_{|\nu| \leq k-1} \frac{x^\nu}{v!} D^\nu K(-y) \right) X_{\{|t|>1\}}(y) \Bigg\} f(y) dy,$$

where

$$K(x) = \omega(x)|x|^{-n}, \int_{S^{n-1}} \omega(x) ds = 0, K_\varepsilon(x) = K(x) X_{\{|t|>\varepsilon\}}(x),$$

the function $\omega(x)$ is homogeneous of power $X_{\{|t|>\varepsilon\}}$ is a characteristically function of the set $\{t \in R^n : |t| > \varepsilon\}, S^{n-1}$ is a unique sphere in the Euclidean space R^n; we assume that for the function $k = 1$ is differentiable $K(x)$ and has bounded, first order partial derivations, while for $K > 1$ the function is $K(x)$ times continuously differentiable on the sphere S^{n-1}.

Theorem. *Let $\alpha > 0, k \in N, k < \alpha + 1$. Then the operator A_k boundedly acts in the space on the space of oscillations type.*

For details see [1].

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Global bifurcation from infinity in some fourth order nonlinear eigenvalue problems with indefinite weight

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We consider the following fourth order nonlinear eigenvalue problem

$$(p(t)u'')'' - (q(t)u')' = \lambda r(t)u + h(t, u, u', u'', u''', \lambda),\ t \in (0, 1), \tag{1}$$

$$\begin{array}{l} u'(0)\cos\alpha - (pu'')(0)\sin\alpha = 0, \\ u(0)\cos\beta + Tu(0)\sin\beta = 0, \\ u'(1)\cos\gamma + (pu'')(1)\sin\gamma = 0, \\ u(1)\cos\delta - Tu(1)\sin\delta = 0, \end{array} \tag{2}$$

where $\lambda \in \mathbb{R}$ is a spectral parameter, $Tu \equiv (pu'')' - qu'$, $p \in C^1([0,1];(0,+\infty))$, $p' \in AC([0,1];\mathbb{R})$, $q(t) \in AC([0,1];[0,+\infty))$, $r(t) \in C([0,1];\mathbb{R})$ and there exist $\xi, \zeta \in [0,1]$ such that $r(\xi)r(\zeta) < 0$, α, β, γ, δ are real constants such that $0 \leq \alpha$, β, γ, $\delta \leq \pi/2$ except the cases $\alpha = \gamma = 0$, $\beta = \delta = \pi/2$ and $\alpha = \beta = \gamma = \delta = \pi/2$. The nonlinear term has the representation $h = f + g$, where $f, g \in C([0,1] \times \mathbb{R}^5; \mathbb{R})$ and satisfy the following conditions: for any $(t,\ u,\ s,\ v,\ w,\ \lambda) \in [0,1] \times \mathbb{R}^5$

$$uf(t,\ u,\ s,\ v,\ w,\ \lambda) \leq 0,\ ug(t,\ u,\ s,\ v,\ w,\ \lambda) \leq 0; \tag{3}$$

there exists constants $M > 0$ such that

$$\left| \frac{f(t,\ u,\ s,\ v,\ w,\ \lambda)}{u} \right| \leq M,\ (t, u, s, v, w, \lambda) \in [0,1] \times \mathbb{R}^5; \tag{4}$$

$$g(t, u, s, v, w, \lambda) = o(|u| + |s| + |v| + |w|), \tag{5}$$

in a neighborhood of $(u, s, v, w) = \infty$ uniformly in $t \in [0,1]$ and in $\lambda \in \Lambda$, for every bounded interval $\Lambda \subset \mathbb{R}$.

The problem (1)-(2) for the case of $f \equiv 0$ is studied in [1]. In this case the linearization of (1)-(2) at $u = 0$ is the linear eigenvalue problem

$$\begin{array}{l}(p(t)u''(t))'' - (q(t)u'(t))' = \lambda r(t)u(t), \ t \in (0,1), \\ u \in B.C.\,, \end{array} \tag{6}$$

where by $B.C.$ we denote the set of boundary conditions (2).

Theorem 1 [1, Theorem 2.1]. *The spectral problem (6) has two sequences of real eigenvalues*

$$0 < \lambda_1^+ < \lambda_2^+ \leq ... \leq \lambda_k^+ \mapsto +\infty \text{ and } 0 > \lambda_1^- > \lambda_2^- \geq ... \geq \lambda_k^- \mapsto -\infty,$$

and no other eigenvalues. Moreover, λ_1^+ and λ_1^- are simple principal eigenvalues, i.e. the corresponding eigenfunctions $u_1^+(t)$ and $u_1^-(t)$ have no zeros in the interval $(0,1)$.

By E we denote be a Banach space $C^3[0,1] \cap B.C.$ with the usual norm $||u||_3 = ||u||_\infty + ||u'||_\infty + ||u''||_\infty + ||u'''||_\infty$, where $||\cdot||_\infty$ is the standard sup-norm in $C[0,1]$.

Let S_k^ν, $k \in \mathbb{N}$, $\nu \in \{-,+\}$, be the set of functions $u \in E$ having nodal properties of eigenfunctions of problem (1)-(2) with $p(t) > 0$, $t \in [0,1]$, and their derivatives, constructed in [2].

For each $k \in \mathbb{N}$, each $\sigma \in \{+,-\}$ and each $\nu \in \{+,-\}$ by $S_{k,\sigma}^\nu$ we denote the set of functions $u \in S_k^\nu$ such that $\sigma \int\limits_0^1 \rho(t)u^2(t)dt > 0$.

Let $J_1^+ = [\lambda_1^+, \lambda_1^+ + d_1^+]$, $J_1^- = [\lambda_1^- - d_1^-, \lambda_1^-]$, where $d_1^\sigma = \frac{\sigma K \int\limits_0^1 \left(u_1^\sigma(t)\right)^2 dt}{\int\limits_0^1 r(t)\left(u_1^\sigma(t)\right)^2 dt}$, $\sigma \in \{+,-\}$.

Lemma 1. *The set of asymptotic bifurcation points of problem (1)-(2) with respect to set $\mathbb{R} \times S_{1,\sigma}^\nu$, $\sigma \in \{-+\}$, $\nu \in \{-+\}$, is nonempty. Moreover, if (λ, ∞) is a bifurcation point of problem (1)-(2) with respect to set $\mathbb{R} \times S_{1,\sigma}^\nu$, then $\lambda \in J_1^\sigma$.*

Let $\mathcal{D}$ be the set of nontrivial solutions of problem (1)-(2). For each $\sigma \in \{+,-\}$ and $\nu \in \{+,-\}$ we define the set $\tilde{D}_1^{\sigma,\nu}$ to be the union of all the components of $\mathcal{D}$ which meet $J_1^\sigma \times \{\infty\}$ through $\mathbb{R} \times S_{1,\sigma}^\nu$. It follows from Lemma 1 that these sets are nonempty. The set $\tilde{D}_1^{\sigma,\nu}$ may not be connected in $\mathbb{R} \times E$, but the set $D_1^{\sigma,\nu} = \tilde{D}_1^{\sigma,\nu} \cup (J_1^\sigma \times \{\infty\})$ is connected in $\mathbb{R} \times E$.

Let $\mathbb{R}^+ = (0, +\infty)$, $\mathbb{R}^- = (-\infty, 0)$.

Theorem 1. *For each* $\sigma \in \{+, -\}$ *let* $\mathcal{P}_1^\sigma = \{(\lambda, u) \in \mathbb{R} \times E : \mathrm{dist}\,\{\lambda, J_1^\sigma\} < \delta_1,\ ||u||_3 > R_1\}$, *where* δ_1 *be the sufficiently small positive number and* R_1 *be the sufficiently large positive number. Then*

1^o. $D_1^{\sigma,\nu} \subset \mathbb{R}^\sigma \times E$ *and* $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma \subset \mathbb{R}^\sigma \times S_{1,\sigma}^\nu$,

and either

2_1^o. $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma$ *is bounded in* $\mathbb{R} \times E$ *in which case* $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma$ *meets* $\mathcal{R}$ *or*

2_2^o. $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma$ *is unbounded in* $\mathbb{R} \times E$. *If additionally* $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma$ *has a bounded projection on* $\mathbb{R}$, *then* $D_1^{\sigma,\nu} \backslash \mathcal{P}_1^\sigma$ *either meets* $J_1^\sigma \times \{\infty\}$ *with respect to the set* $\mathbb{R}^\sigma \times S_{1,\sigma}^{-\nu}$, *or meets* $J_k^\sigma \times \{\infty\}$ *for some* $k \neq 1$, *where* $J_k^\sigma \times \{\infty\}$ *is a bifurcation interval of problem (1)-(2) which surrounds an eigenvalue* λ_k^σ *of the linear problem (6).*

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Limit theorems for the random walks describes by the generalization of autoregressive process of order one $(AR(1))$

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Let (Ω, F, P) be a probability space, and $\{\xi_n,\ n \geq 1\}$ be a sequence of independent random variables with $E\xi_n = 0$ and $E\xi_n = \sigma_n^2$. Define the sequence random variables $\{X_n\}$ by

$$X_n = \theta_0 X_{n-1} + \xi_n$$

for some fixed number $\theta_0 \in (-\infty, \infty)$, where initial value X_0 is independent of $\{\xi_n\}$ [1].

Set

$$A_n = \sum_{i=1}^{n} \frac{X_i X_{i-1}}{\sigma_i^2}.$$

In this work we prove following theorem.

Theorem. Let $\{\xi_n,\ n \geq 1\}$ be a sequence of independent random variables with $E\xi_n = 0$ and $E\xi_n^2 = 1$. Suppose that $\sum_{n=1}^{\infty} E\left(\xi_n^2 \wedge 1\right) = \infty$ and $|\theta_0| < 1$, $EX_0^2 < \infty$. Then as $n \to \infty$

$$\frac{A_n}{n} \xrightarrow{a.s.} \frac{\theta_0}{1-\theta_0^2},$$

$$\sqrt{n}\left(\frac{A_n}{n} - \frac{\theta_0}{1-\theta_0^2}\right) \xrightarrow{d} N(0, \alpha_2), \quad where \quad \alpha_2 = \frac{1}{1-\theta_0^2}.$$

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Initial-boundary value problem for systems of wave equations with nonlinear boundary dissipation and with a nonstandard interior nonlinear source

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Consider the initial - boundary value problem:

$$u_{i_{tt}} - u_{i_{xx}} = f_i(x, u_1, u_2), 0 < x < l,\ t > 0,\ i = 1, 2, \tag{1}$$

$$u_i(0, t) = 0,\ t > 0,\ i = 1, 2, \tag{2}$$

$$u_{i_x}(l, t) + |u_{i_t}(l, t)|^{r_i - 1} u_{i_t}(l, t) = 0,\ t > 0,\ i = 1, 2, \tag{3}$$

$$u_i(0, x) = u_{i0}(x),\ u_{i_t}(0, x) = u_{i1}(x),\ 0 < x < l,\ t > 0,\ i = 1, 2, \tag{4}$$

where

$$f_i(x, u_1, u_2) =$$

$$= a(x) |u_1 + u_2|^{p_1(x)+p_2(x)} (u_1+u_2)+b_i(x) |u_1|^{p_1(x)+(-1)^i} |u_2|^{p_2(x)-(-1)^i} u_i, i = 1, 2.$$

Suppose that the

1. $a(\cdot) \in C([0, l]; R)$;

2. $b_i(x) = \lambda(x)(p_i(x) + 1),\ 0 \le x \le l,\ i = 1, 2$ where $\lambda(\cdot) \in C([0, l]; R)$, $p_i(x),\ i = 1, 2$ are real - valued functions,

$$1 < \min_{0 \le x \le l} p_i(x) = p_{i1}, \quad \max_{0 \le x \le l} p_i(x) = p_{i2}$$

and the following inequality

$$|p_i(x) - p_i(y)| \le \frac{A_i}{\log|x - y|},\ A_i > 0,$$

holds for any $x, y \in [0, l],\ \ |x - y| < \delta$;

3. $r_1,\ r_2 \ge 1$;

4. $u_{i0}(\cdot) \in {}_0H^1,\ u_{i1}(\cdot) \in L_2(0, l),\ i = 1, 2$ where ${}_0H^1 = \{v : v \in H^1,\ v(0) = 0\}$.

We define the energy function

$$E(t)=\frac{1}{2}\sum_{i=1}^{2}\left[\|u_{i_t}(t,\cdot)\|_2^2+\|u_{i_x}(t,\cdot)\|_2^2\right]-G(u_1(t,\cdot),u_2(t,\cdot)),$$

where

$$G(u_1,u_2)=\int_0^l \frac{a(x)}{p_1(x)+p_2(x)+2}\left|u_1(x)+u_2(x)\right|^{p_1(x)+p_2(x)+2}dx+$$

$$+\int_0^l \lambda(x)\left|u_1(x)\right|^{p_1(x)+1}\left|u_2(x)\right|^{p_2(x)+1}dx.$$

Theorem 1. Assume that conditions 1-4 are satisfied. Then there exists a $T'\in(0,T]$, such that problem (1)-(4) has a unique solution $\{u_1(\cdot),u_2(\cdot)\}$, where $u_i(\cdot)\in C\left([0,T'];{}_0H^1\right), u_{i_t}(\cdot)\in C\left([0,T'];L_2(0,l)\right), u_{i_t}(l,t)\in L^{r_i+1}(0,T'),\ i=1,2$ and has the identity

$$E(t)+\sum_{i=1}^{2}\int_0^t \left|u_{i_\tau}(l,\tau)\right|^{r_i+1}d\tau=E(0).$$

Theorem 2. Assume that conditions 1-4 are satisfied, additionally suppose that

$$a(x)\geq a_0,\ 0\leq x\leq l,\ a_0\in(0,+\infty),$$

$$\lambda(x)\geq\lambda_0,\ 0\leq x\leq l,\ \lambda_0\in(0,+\infty),$$

$$1<\min\{r_1,r_2\}\leq\max\{r_1,r_2\}<\frac{p_{11}+p_{21}}{2}+1$$

and

$$E(0)<0.$$

Then the local solutions defined by Theorem 1 blows - up in finite time.

When $p_2(x)=p_1(x)$ the problem (1)-(4) studied in [1], and in the case when $p_2(x)=p_1(x)=p$, where p some constant, the problem of blows – up of solutions in finite time. to problem (1)-(4) has studied in the works [2, 3].

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About the commentaries of Nasireddin Tusi to the work of Archimedes "About the ball and the cylinder"

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The comments of Nasireddin Tusi to the first part of the first book "About the ball and the cylinder", which includes Theorems 1-16, which are devoted to the definition of the lateral surface of the cone and the cylinder, are considered.

In Theorem 3, N.Tusi gives a proof of the relation about the inequality of the sides of an equilateral figure circumscribed and inscribed near a circle. N.Tusi deftly draws a straight line that selects an equal angle to a comparable angle in a right-angled triangle in order to obtain similar triangles with equal ratios of similar sides from three equal angles, and to derive the ratio of inequality of ratios of the sides of triangles using the greater slope to the same perpendicular.

In Theorem 8, N.Tusi gives a proof of the theorem on three perpendiculars, the first proof of which the researchers address to N.Tusi

The theorem on three perpendiculars was known to mathematicians of the Near and Middle East, its proof is available in the "Treatise on the complete quadrilateral" by N. Tusi. In Europe, this theorem was first formulated by Louis Bertrand (1731-1812) and proved in Legendre's "Elements of Geometry" (1794). Legendre's proof is reproduced in Kiselev's textbook.

In Theorem 11, N. Tusi makes a very important comment about the inadmissibility of applying the assertion of the theorem in the course of the proof. "The statement of the theorem is used in the proof." This is repeated many times in the text. This is an unacceptable contradiction of Archimedes.

Nasireddin Tusi also makes the most important remark to this theorem: "Under the assumptions of Archimedes: "Of the surfaces that have a common boundary located on a plane, the plane will be the smallest.

Two surfaces that have a common boundary located on a plane will always be unequal if both of them are convex in the same direction and one of them is completely enclosed by another surface and a plane containing the common boundary, while the enclosing surface will be smaller.""

The scattering problem for the hyperbolic system of six first order equations on a semi-axis with three given incident waves

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We consider the following hyperbolic system of six first order equations on the semi-axis $x \geq 0$:

$$\xi_i \frac{\partial \psi_i(x,t)}{\partial t} - \frac{\partial \psi_i(x,t)}{\partial x} = \sum_{j=1}^{6} c_{ij}(x,t) u_j(x,t), \tag{1}$$

$i = \overline{1,6}$, $-\infty < t < +\infty$, $\xi_1 > \xi_2 > \xi_3 > 0 > \xi_4 > \xi_5 > \xi_6$, where $\{\psi_1(x,t), \cdots \psi_6(x,t)\}$ is desired function, $c_{ij}(x,t)$ $(i,j = \overline{1,6})$ are measurable by x and t complex-valued functions satisfying condition:

$$|c_{ij}(x,t)| \leq c \left[(1+|x|)(1+|t|)\right]^{-1-\varepsilon} \tag{2}$$

$c > 0, \varepsilon > 0$ -constants, and besides $c_{ii}(x,t) \equiv 0$, $i,j = \overline{1,6}$.

Any substantially bounded solution of the system (1) with coefficients satisfying (2) assumes the following asymptotic representations on the semi-axis $x \geq 0$:

$$\psi_k(x,t) = a_k(t + \xi_k x) + 0(1), k = 1,2,3,$$
$$\psi_k(x,t) = b_k(t + \xi_k x) + 0(1), k = 4,5,6, \ x \to +\infty \tag{3}$$

where the functions $a_k(s) \in L_\infty(R), R = (-\infty, +\infty)$, $k = 1,2,3$ define incident waves, and $b_k(s) \in L_\infty(R), k = 4,5,6$ define scattered ones.

We consider three problems. Each problem is to find the solution $\psi(x,t) = \{\psi_1(x,t), \cdots, \psi_6(x,t)\}$ of the system (1) satisfying one of the following conditions:

$$I.\varphi_2^1(o,t) = H_1 \varphi_1^1(o,t), \tag{4}$$

$$II.\varphi_2^2(o,t) = H_2 \varphi_1^2(o,t), \tag{5}$$

$$III.\varphi_2^3(o,t) = H_3\varphi_1^3(o,t), \tag{6}$$

where $\varphi(x,t) = \{\varphi_1(x,t), \varphi_2(x,t)\}$, $\varphi_1(x,t) = \{\psi_1(x,t), \psi_2(x,t), \psi_3(x,t)\}$, $\varphi_2(x,t) = \{\psi_4(x,t), \psi_5(x,t), \psi_6(x,t)\}$, $H_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $H_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$,
$H_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Joint consideration of these three problems will be called the scattering problem for the system (1) on a semi-axis.

Theorem 1. *Let the coefficients $c_{ij}(x,t)$, $i,j = \overline{1,6}$ of the system (1) satisfy the conditions (2). Then there is a unique solution of the scattering problem for the system (1) on the semi-axis $x \geq 0$ with given incident waves $a_1(s), a_2(s), a_3(s) \in L_\infty(R)$.*

Each solution assumes the following asymptotics in space $L_\infty(R)$

$$\psi_i^k(x,t) = b_i^k(t + \xi_i x) + 0(1), x \to +\infty, \; i = 3,4,5; \; k = 1,2,3. \tag{7}$$

Thus we can define operators S^1, S^2, S^3 converting $(a_1(s), a_2(s), a_3(s)) \in L_\infty(R, C^3)$ into $(b_4^k(s), b_5^k(s), b_6^k(s)) \in L_\infty(R, C^3)$, $k = 1,2,3$. Operator $S = (S^1, S^2, S^3)$ is called the scattering operator for the system (1) on a semi-axis.

Note that the inverse scattering problem for the hyperbolic system of $2n$ equations with n incident and n scattered waves when some coefficients equal zero was studied in [1].

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Quality properties of solutions of nonuniformly degenerated second order elliptic equations

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The paper deals with quality properties of nonuniformly degenerate nondivergent second order elliptic equations with discontinuous coefficients. Let E_n be n-dimensional Euclidean space of the points $x = (x_1, ..., x_n) n \geq, 3$ $D \subset E_n$ be a bounded domain with $\partial D \in C^2$ and $0 \in \overline{D}$. In D we consider the following elliptic equation

$$Lu = \sum_{i,j=1}^{n} a_{ij}(x) u_{ij} + \sum_{i=1}^{n} b_i(x) u_i + c(x) u = f(x), \tag{1}$$

Assume that, $\|a_{ij}(x)\|$ is a real symmetric matrix, and for $x \in D$ the following condition is fulfilled:

$$\sup_D \left[\sum_{i,j=1}^{n} \frac{a_{ij}^2(x)}{\lambda_i(x)\lambda_j(x)} \Bigg/ \left(\sum_{i=1}^{n} \frac{a_{ii}(x)}{\lambda_i(x)} \right)^2 \right] < \frac{1}{n - e^2}. \tag{2}$$

$$\inf_D \sum_{i,j=1}^{n} \frac{a_{ii}(x)}{\lambda_i(x)} = \gamma, \;\; e = \inf_D \sum_{i,j=1}^{n} \frac{a_{ii}(x)}{\lambda_i(x)} \Bigg/ \sup_D \sum_{i=1}^{n} \frac{a_{ii}(x)}{\lambda_i(x)} \tag{3}$$

$$|B(x)| \leq b_0, i = 1, ..., n; -c_0 \leq c(x) \leq 0, \quad x \in D, \tag{4}$$

where $|B(x)| = \left(\sum\limits_{i,j=1}^{n} \frac{b_i^2(x)}{\lambda_i(x)} \right)^{1/2}$, $\gamma \in (0, 1]$, b_0 and c_0 are non-negative constants, $\lambda_i(x) = \left(\frac{\omega_i^{-1}(\rho(x))}{\rho(x)} \right)^2$, $\rho(x) = \sum\limits_{i=1}^{n} \omega_i(|x_i|)$, $u = u(x)$, $u_i = \frac{\partial u}{\partial x_i}$, $u_{ij} = \frac{\partial^2 u}{\partial x_i \partial x_j}$, $i, j = 1, ..., n$. Assume that $\omega_i(t)$, $i = 1, ..., n$ are continuous, positive, monotonically increasing on $[0, diamD]$ functions, $\omega_i(0) = 0$, $\omega_i^{-1}(t)$, $i = 1, ..., n$ is a function inverse to $\omega_i(t)$. Furthermore there exist such numbers $\alpha > 1, \beta > 1,\ \eta > 0$, $A > 0$ and $q > n$ that

$$\alpha\omega_i(t) \le \omega_i(\eta t) \le \beta\omega_i(t),\ i = 1, ..., n,\ t \in 0, (diamD), \tag{5}$$

$$\left(\frac{\omega_i^{-1}(t)}{t}\right)^{q-1} \int\limits_0^{\omega_i^{-1}(t)} \left(\frac{\omega_i(\tau)}{\tau}\right)^q d\tau \le At, \quad i = 1, ..., n. \tag{6}$$

The condition (2) is called the Cordes elliptic condition. It is understood to within nonsingular linear transformation. Note that the condition (2) coincides with the classic Cordes condition (i.e. for $\lambda_i \equiv 1, e = 1$).

Variational optimality conditions in control problems of hybrid distributed parameter systems and applications

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We consider a special class of optimal control problems for linear systems of first-order hyperbolic equations:

$$\frac{\partial x}{\partial t} + A(s,t)\frac{\partial x}{\partial s} = \Phi(s,t)x + f(y(t),s,t),$$

$x = x(s,t)$ is n –dimensional vector-function. In this problems, the function $y(t)$ in the right-hand side of the hyperbolic system and/or boundary conditions are determined from controlled systems of ordinary differential equations.

In [1] a variant of a linear objective functional is considered. Here we study a case of a linear-quadratic cost functional. The problem is reduced to an optimal control problem for a system of ordinary differential equations. The reduction is based on non-classic exact increment formulas of the cost functional. The results are formulated as variational optimality conditions. Such problems arise in the simulation of some processes of chemical technology and population dynamics.

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Some ineqalities related to distribution functions of negative eigen-values of differential operators on the semi-axis

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In space $L_2[0,\infty)$ consider the differential operator L generated by the expression

$$l(y) = -y'' - q(x)y \tag{1}$$

and the boundary condition

$$y'(0) = 0. \tag{2}$$

Assume that the functions $q(x)$ satisfies the following conditions:

1) the function $q(x)$ is continuous, monotonically decreasing and positive on the interval $[0,\infty)$

2) $\lim\limits_{t\to\infty} q(x) = 0$

3) for $k_0 \in \left(0, \frac{2}{9}\right)$ and $\eta > 0$ $\lim\limits_{x\to\infty} = \left[q(x)x^{K_0+\eta}\right]^{-1} = 0.$

The operator L is self-adjoint lower bounded and the negative part of its spectrum is discrete. Let the negative eigenvalues of operator be $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \leq \ldots$. In this abstract we study the negative spectrum of a second order differential operator L on the semi-axis. Discreteness conditions of the negative spectrum are obtained and some estimates are proved for the distributions function of negative eigenvalues.

On well-defined solvability of the dirichlet problem for a second order elliptic partial operator-differential equation in Hilbert space

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In Hilbert space H we consider the following boundary value problem for a second order elliptic partial operator-differential equation

$$Lu = -\sum_{k=1}^{n} a_k \frac{\partial^2 u(x)}{\partial x_k^2} + \sum_{k=1}^{n} R_k \frac{\partial u(x)}{\partial x_k} + Tu(x) + C^2 u(x) = f(x),\ x \in R_+^n 1 \quad (1)$$

$$u(x)|_{x_n=0} = 0, \quad (2)$$

where $(x_1, x_2, \ldots, x_{n-1}) \in R^{n-1}, x_n \in R_+^n,\ x = (x_1, x_2, \ldots, x_{n-1}, x_n) \in R_+^n$, $f(x)$ and $u(x)$ are the vector functions determined in almost everywhere in R_+^n, with the values in H, and the operator coefficients of equation (1) satisfy the following conditions:

1. The scalar numbers $a_k > 0$, $k = 1, 2, \ldots, n$,

2 C is a self-adjoint positive-definite operator in H,

2. The operators $Q_k = R_k C^{-1}\,(k = 1, 2, \ldots, n)$ and $F = TC^{-2}$are bounded in H.

We have the following theorem.

Theorem *Let operator coefficients of equation (1)-(2) satisfy the conditions 1)-3) and the following condition be fulfilled:*

$$q = \frac{1}{2} \sum_{k=1}^{n} \frac{\|Q_k\|}{\sqrt{a_k}} + \|F\| < 1$$

.

Then boundary value problem (1)-(2) well defined solvable in space $W_2^2\left(R_+^n; H\right)$.

Description of inhomogeneous food medium by rheological models

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Mechanical and hydromechanical processes occurring in installations (machines and apparatus) used in food industry are based on rheological laws of deformable food medium of solid mechanics [1]. Therefore since the process of separation of inhomogeneous liquid form food system is a step where hydromechanical process there arise a problem to study this process at each process.

Under liquid non-homogeneous food systems we understand turbid semi-dispersed systems consisting of colloidal substances, coarse and finely dispersed particles. To dilute such systems, the following processes are performed sequentially: dilution process; filtration process; centrifugation process; separation process. Each of these processes serves to separate the fractions that make up the liquid homogeneous food system and is carried out on appropriate equipment.

The main factors influencing the filtration rate of suspensions in equipment with a filtration filter are: pressure drop, thickness of the sediment layer in the filter, structure and nature of the sediment, composition, viscosity and temperature of the suspension.

To increase the velocity of liquid food medium filtration in porous medium. The existing methods and technologies to increase its viscosity are improved and replaced by new ones. The efficiency of these methods and technologies is related to change in physical and mechanical properties of high viscosity liquid food systems. This shows that description of new modules with into account interaction of molecules and atoms.

It is known that viscous-elastic liquid and amorphous body media are described by linear viscous-elastic (two-element rheological models) Maxwell and Voight models. These models consist of sequentially and parallel connected elastic and viscous elements. These models with the known solutions mathematically are described by the following relations.

The Maxwell model:

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \mu\sigma, \tag{1}$$

The Voight model:

$$\sigma = E\varepsilon + \mu\dot{\varepsilon}\ , \tag{1}$$

here, ε is total deformation, $\varepsilon_i$is deformation in each element $(i = 1; 2)$, σ is stress, $\sigma_i(i = 1, 2)$ is a stress in each element, E is an elasticity modules, μ is an elasticity factor. The doth on the symbol shows that the quantity is differentiable with respect to time.

Both models described by rheological models and taking into account metal nanoparticles, the stress-strain state of the medium corresponding to each model studies. In the new models, the nature of the change in parameters and their values are described in the framework of the hypothesis of the interaction of metal nanoparticles with atoms of the medium.

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Emergence and history of complex analysis

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The "Theory of Functions of Complex Variable" (TFCV) or, otherwise, Complex Analysis is one of the disciplines considered classical and mandatory today within the framework of educational programs of higher education for the training of mathematicians, physicists, engineers, teachers, etc.

The beginning of the development of the TFCV is usually counted from the works of Gerolamo Cardano, in particular with "Artis magnae sive de regulis algebraicis" (Great Art, or On Algebraic Rules, 1545). The significance of this book in the history of mathematics already in the 20 th century was described by Felix Klein, who said "This highly valuable work contains the embryo of modern algebra that goes beyond ancient mathematics."

From the works of Leonhard Euler (in particular, "Introduction to the analysis of infinitesimal", 1748), mathematicians use both a complex variable and the first letter of the word "imaginaire" (imaginary) to indicate the root of -1. Another scientist who used the elements of TFCV to solve hydrodynamics problems was J.L. Lagrange, who published mathematical analysis courses in Paris "Theory of Analytical Functions" (1797). In the same year, Caspar Wessel, in his work "On the Analytical Representation of Directions", presented a visual geometric interpretation of complex numbers set, which gave additional opportunities for the active development of complex analysis.

After the publications of 1826-1850 by Augustin Louis Cauchy, disparate facts on the differential and integral calculus of the functions of a complex variable were combined and systematized, the theory of deductions was actually created, which later became a powerful tool in the applications of TFCV, integral formulas were derived, decomposition of the function of a complex variable into power series was investigated, the foundations of the theory of analytical functions of many variables were laid. From this moment, and especially after the work of Bernhard Riemann and Karl Weierstra, TFCV as a scientific discipline enters the stage of extensive expansion, mathematicians around the world create functional branches of applications of complex analysis - and the development of a full-fledged educational discipline begins.

Back in the 1820s, Cauchy's lectures on complex analysis at the Polytechnic School were attended by the Russian mathematician M.V. Ostrogradsky, but at that time and until the end of the 19 th century, it is usually considered as part of mathematical analysis. In the domestic higher school, this tradition lasts until the middle of the twentieth century, for example, in the classic textbook by V.I. Smirnov "Course in Higher Mathematics" (1930) TFCV is still considered as an integral part of general courses, and only by the 1950s were general traditions of teaching the discipline formed and classical textbooks were published - A.I. Markushevich "Theory of analytical functions" and "Essays on the history of the theory of analytical functions" and I.I. Privalov "Introduction to the theory of functions of complex variable". After that, in parallel with the development of comprehensive analysis as a scientific discipline, a whole range of classical university textbooks for mathematics students, physicists and students of engineering specialties is published under the authorship of B.A. Fuks, B.V. Shabat, M.A. Lavrentiev and others. By this time, the basic training course "TFCV" had already been formed, as may be evidenced by the explanation of the author (B.V. Shabat) of the textbook "Introduction to complex Analysis" in 1969: "The first part, dedicated to the functions of one variable, contains material from a mandatory university course. The second part deals with the functions of several variables and contains the material of the main special course."

From that moment on, we can talk about the "canonization" of the TFCV discipline, which has become a full-fledged part of the educational process. In recent years, the work of mathematicians in this industry has been devoted primarily to the development of effective applications of theory and, accordingly, textbooks to accompany the relevant special courses. A separate area of activity in this direction is the development of the most universal teaching aids adapted to different levels of teaching, as well as appropriate electronic educational resources and electronic manuals.

Asymptotics of eigenvalue distribution of one class of selfadjoint extensions

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Consider the eigenvalue problem :

$$ly = y^{IV} + Ay(t) = \lambda y(t),\ y(0) = y''(0) = 0$$

$$-y'''(1) = \lambda Q_1 y(1), y''(1) = \lambda Q_2 y'(1)$$

Operator corresponding to that problem denote by L_0^1 which is selfadjoint and discrete. Now we search the asymptotics of eigenvalues of that operator.

Take here for simplicity of calculations $Q_1 = Q_2 = A^\alpha$ and $0 < \alpha < \frac{1}{2}$.

Theorem. The algebraic multiplicity of eigenvalues $\lambda_{k,j}$ of the operator L_0^1 is two and the following asymptotic formula is true:

$$\lambda_{k,j} = \gamma_k + z_{k,j}^4, z_{k,j} \sim \left\{ \begin{array}{c} \pi j + \frac{\pi}{4} + O\left(\frac{1}{k}\right), \\ i\left(\pi j + \frac{\pi}{4} + O\left(\frac{1}{k}\right)\right) \end{array} \right.$$

and $\lambda_{k,j} = \gamma_k + \eta_{k,j}^4$, where $\eta_{k,j}$ are small in modulus zeros of characteristic determinant, γ_k are eigenvalues and φ_k are orthonormal basis formed by eigenvectors of A.

Lemma. For distribution function $N(\lambda) = \sum\limits_{\lambda_{n<\lambda}} 1$ of eigenvalues of operator L_0^1 the next relation is valid $N(\lambda) \sim C_1 \lambda^{\frac{4+\alpha}{4\alpha}}$ for sufficiently large λ.

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The boundedness of fractional integral operators in generalized weighted Morrey spaces on Heisenberg groups

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We study Spanne-Guliyev and Adams-Guliyev type boundedness of the fractional integral operator I_α, $0 < \alpha < Q$ on Heisenberg group $\mathbb{H}_n$ in the generalized weighted Morrey spaces $M_{p,\varphi}(\mathbb{H}_n, w)$, where Q is the homogeneous dimension of $\mathbb{H}n$.

Let $f \in Ł_1^{loc}\mathbb{H}_n$. The fractional integral operator I_α is defined by

$$I_\alpha f(u) = \int_{\mathbb{H}_n} \frac{f(v)\, dV(v)}{\left|u^{-1}v\right|^{Q-\alpha}}, \qquad 0 < \alpha < Q,$$

where Q is the homogeneous dimension of the Heisenberg group $\mathbb{H}_n$ and $|B(u,r)|$ is the Haar measure of the $\mathbb{H}_n$- ball $B(u,r)$.

We obtained the following Guliyev weighted local estimate (see, for example, [2, 3] in the nonweighted case).

Theorem 1. *Let* $1 \le p < q < \infty$, $0 < \alpha < \frac{Q}{p}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{Q}$, *and* $\omega \in A_{p,q}(\mathbb{H}_n)$. *Then, for* $1 < p < q < \frac{Q}{\alpha}$, *the inequality*

$$\|I_\alpha f\|_{L_{q,w^q}(B(u,r))} \lesssim w^q(B(u,r))^{\frac{1}{q}} \int_r^\infty \|f\|_{L_{p,w^p}(B(u,t))} w^q(B(u,t))^{-\frac{1}{q}} \frac{dt}{t}$$

holds for any balls $B(u,r)$ *and for all* $f \in L_{p,w}^{loc}(\mathbb{H}_n)$.

Moreover, for $p = 1$ *the inequality*

$$\|I_\alpha f\|_{WL_{q,w^q}(B(u,r))} \lesssim w^q(B(u,r))^{\frac{1}{q}} \int_r^\infty \|f\|_{L_{1,w}(B(u,t))} w^q(B(u,t))^{-\frac{1}{q}} \frac{dt}{t} \qquad (1)$$

holds for any balls $B(u,r)$ *and for all* $f \in L_{1,w}^{loc}(\mathbb{H}_n)$.

The following Spanne-Guliyev type result on the space $M_{p,\varphi}(w)$ is valid.

Theorem 2. *Let* $1 \le p < q < \infty$, $0 < \alpha < \frac{Q}{p}$, $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{Q}$, $w \in A_{p,q}(\mathbb{H}_n)$, *and* (φ_1, φ_2) *satisfy the condition*

$$\int_r^\infty \frac{ess \inf_{t<s<\infty} \varphi_1(u,s)\, w^p B((u,s))^{1/p}}{w^q(B(u,t))^{1/q}} \frac{dt}{t} \le C\varphi_2(u,r)$$

where C *does not depend on* u *and* r. *Then the operator* I_α *is bounded from* $M_{p,\varphi_1}(w^p)$ *to* $M_{q,\varphi_2}(w^q)$ *for* $p > 1$ *and from* $M_{1,\varphi_1}(w)$ *to* $WM_{q,\varphi_2}(w^q)$ *for* $p = 1$. *Moreover, for* $p > 1$

$$\|I_\alpha f\|_{M_{q,\varphi_2}(w^q)} \lesssim \|f\|_{M_{p,\varphi_1}(w^p)},$$

and for $p = 1$

$$\|I_\alpha f\|_{WM_{q,\varphi_2}(w^q)} \lesssim \|f\|_{M_{1,\varphi_1}(w)}.$$

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Tables in the mathematical and logical works of N. Tusi

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The phenomenon of the table appeared in ancient times, and at present the tabular method - a method of structuring information in a graphically visual form - is a research tool in all areas of knowledge. The table allows you to cover the data entered in it as a single system, the expressiveness of such a representation helps to identify connections or quickly achieve a cognitive effect.

As even a superficial acquaintance with the works of Nasireddin Tusi shows, he widely used the tabular method. For a more detailed study of his methodology, we limited ourselves to his works "Collection on arithmetic with the help of a board and dust", "Treatise on the complete quadripartite" and the logical treatise "Fundamentals of Knowledge Acquisition". It is characteristic that in all treatises the tables are given in matrix form.

There are six large tables in the "Collection on arithmetic using the board and dust." In addition to the so-called "Pythagorean" multiplication table (corresponding to the modern one) in the decimal system, a multiplication table in the sexagesimal system is given in the part devoted to "fractions used by astronomers." There are also tables of division and extraction of roots of the second and third degrees. In the part devoted to arithmetic operations, a tabular method of multiplication is given.

Note that arithmetic operations are accompanied by a kind of mini-tables. A remarkable fact is the letter designation of the degrees, which is not noted either among the predecessors of Tusi, nor among subsequent Islamic mathematicians. Tables of multiplication and division of powers are given for general and for specific values. Tusi places the table of binomial coefficients in the form of a triangle, which is now "Pascal's Triangle"

In the "Treatise on the complete quadripartite" Tusi makes extensive use of the graph-matrix method. In the first book on the properties of compound relations, he proves the corresponding theorems by tabulating data and results. At the same time, he notes that two tables were compiled by "persons who own this art" (which speaks of Tusi's scientific ethics). In the second book "Treatise on the complete quadripartite" on "the existing relations in a spherical complete quadripartite", Tusi uses 2 tables that reveal the dependence of the properties of the "non-participating (auxiliary) triangle" on its angles and the dependence on its distance from the "non-participating circle plane". In the fifth book, when determining the methods by which one can find "unknown quantities in a spherical triangle from known ones", a table is used, modal, affirmative or negative statements are placed in the cells of the results.

The logical treatise "Fundamentals of Knowledge Acquisition" contains 43 tables, including Psela's logical square, truth tables for categorical, indefinite, general and particular judgments, tables of varieties of conditional propositions (affirmative and negative), as well as tables related to inferences and syllogisms. Such systematization in the form of tables is not found in previous logicians. This systematization is determined by Tusi's desire for the mathematization of logic, which he himself speaks of in this treatise.

Unfortunately, Tusi's role in the development of the tabular method has gone unnoticed

On the dispersion of axisymmetric waves in the pre-strained highly elastic plate loaded by compressible inviscid fluid

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The current state of research on the dynamics of the plate + fluid system is detailed in the papers [1-3]. Accordingly, the dispersion of axisymmetric waves in the finite pre-strained highly elastic plate loaded by a compressible fluid has not been investigated so far. The present work undertakes some attempts in this field, and the novelty of the work consists in.

2.Mathematical formulation of the problem.

Consider the hydro-elastic system shown schematically in Fig. 1 and associate the cylindrical coordinate system $Or\theta z$ with the upper surface plane of the plate and assume that before the

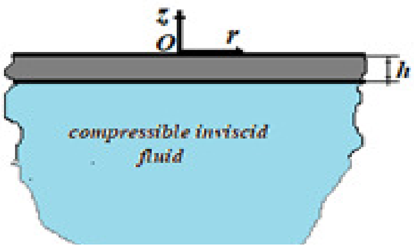

Fig. 1. Sketch of the hydro-elastic system under consideration

axisymmetric waves propagate in the system, the plate is stretched (or compressed) by homogeneous, uniformly distributed radial forces acting at infinity and these forces cause the initial strain state in the plate determined by the displacements $u_r^0 = (\lambda_1 - 1)r, u_\theta^0 = 0$, $u_z^0 = (\lambda_3 - 1)z$, where λ_1 and λ_3 are constants and are called elongation parameters. Assume that the material of the plate is highly elastic, for which the elasticity relations are determined by the harmonic potential [4]. Within the foregoing assumptions, we investigate the dispersion of the axisymmetric waves propagating in the plate +fluid system under consideration, and for this study, we use the Lagrangian coordinates $r'$and z'in the coordinate system $O'r'\theta'z'$, which refers to the aforementioned initial state and is determined by the coordinates r and z by the expressions $r' = \lambda_1 r$, $z' = \lambda_3 z$, according to which, the thickness of the plate becomes h' where $h' = \lambda_3 h$.

Thus, we write the following linearized field equations for the motion of the plate.

$$\frac{\partial Q_{r'r'}}{\partial r'} + \frac{\partial Q_{z'r'}}{\partial z'} + \frac{1}{r'}\left(Q_{r'r'} - Q_{\theta'\theta'}\right) = \rho'\frac{\partial^2 u_{r'}}{\partial t^2}, \frac{\partial Q_{r'z'}}{\partial r'} + \frac{\partial Q_{z'z'}}{\partial z'} + \frac{1}{r'}Q_{r'z'} = \rho'\frac{\partial^2 u_{z'}}{\partial t^2}.$$

$$Q_{r'r'} = \omega'_{1111}\frac{\partial u_{r'}}{\partial r'} + \omega'_{1122}\frac{u_{r'}}{r'} + \omega'_{1133}\frac{\partial u_{z'}}{\partial z'}, \ldots, Q_{z'r'} = \omega'_{3113}\frac{\partial u_{r'}}{\partial z'} + \omega'_{3131}\frac{\partial u_{z'}}{\partial r'}, \quad (1)$$

The meaning of the notation used in (1) is given in [4].

The flow of the fluid is described by the following linearized Navier-Stokes equations.

$$\frac{\partial V_{r'}}{\partial t} = -\frac{1}{\rho_{10}}\frac{\partial p'}{\partial r'}, \frac{\partial V_{z'}}{\partial t} = -\frac{1}{\rho_{10}}\frac{\partial p'}{\partial z'}, p'_1 = a_0^2\rho'_1, a_0^2 = \left(\frac{\partial p_1}{\partial \rho_1}\right)_0,$$

$$\frac{\partial \rho'_1}{\partial t} + \rho_{10}\left(\frac{\partial V_{r'}}{\partial r'} + \frac{V_{r'}}{r'} + \frac{\partial V_{z'}}{\partial z'}\right) = 0. \quad (2)$$

In (2), we use the following notation:

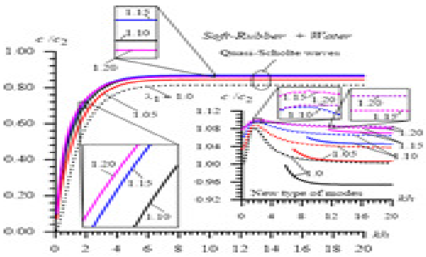

Fig.2 The influence of the initial strains in the plate on the dispersion curves of the lowest mode

The equations are supplied the following compatibility and contact conditions.

$$Q_{z'z'}\big|_{z'=0} = 0,\ Q_{z'r'}\big|_{z'=0} = 0,\ Q_{z'z'}\big|_{z'=-h'} = -\left.p'_1\right|_{z'=-h'},\ Q_{z'r'}\big|_{z'=-h'} = 0,$$

$$\left.\frac{\partial u_{z'}}{\partial t}\right|_{z'=-h'} = \left.V_{z'}\right|_{z'=-h'}, \left.V_{z'}\right|_{z'=-h'-h_d} = 0. \qquad (3)$$

This completes mathematical formulation of the problem $g(r,z,t) = g_1(z)J_0(kr)e^{i\omega t}$ for all the values we are looking for, where $J_0(kr)$ is the Bessel function of the first kind with zero order, k is the wave number, and ω is the wave frequency. Substituting these representations into the above equations, we obtain the corresponding differential equations for determining the $g_1(z)$ type functions.

Applying the usual solution procedure we obtain the dispersion equation in terms of k and ω. The solution of this equation is done numerically using PC programs based on the "bi-section" method.

2.On the numerical result

Numerical results are obtained for the quasi-Scholte waves and some new lowest modes under various problem parameters, focusing on the influence of the parameter λ_1 on the dispersion curves. Fig. 2 shows these dispersion curves as an example for the case where water is assumed to be the liquid and the material of the plate is soft rubber. From these dispersion curves, it follows that the initial elongation of the plate leads, in general, to an increase in the wave propagation velocity.

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On the absence of global solutions of inhomogeneous evolution semilinear inequalities with a biharmonic operator in the main part

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In the half-space $D = (0, +\infty) \times R^n$ consider the following inequalities

$$|u|_t \geq \Delta^2[|u|^\sigma u] + |u|^q + \omega(x) \tag{1}$$

$$u_t \geq \Delta^2[|u|^\sigma u] + |u|^{q-1} u + \omega(x), \tag{2}$$

where $n > 4, q > 1, \sigma \geq 0$ and $\omega : R^n \to [0, +\infty)$ is a function which belongs to $L_{1,loc}(R^n)$.

Definition. By a solution of (1), (2) on D we understand a function $u(x, t)$ which belongs to the space $L_{q,loc}(D) \bigcap L_{1+\sigma,loc}(D)$ and satisfies the integral inequality

$$\int_D [-|u| \varphi_t - u |u|^\sigma \Delta^2\varphi] dtdx \geq \int_D [|u|^q + \omega(x)]\varphi dtdx$$

$$\left(\int_D [-u\varphi_t - u |u|^\sigma \Delta^2\varphi] dtdx \geq \int_D [u |u|^{q-1} + \omega(x)]\varphi dtdx\right)$$

for any nonnegative function $\varphi \in C_0^\infty(D)$.

The main results are the following theorems.

Theorem 1. Let $1 + \sigma < q \leq \frac{n(\sigma+1)}{n-4}$, and $\omega(x)$ be nontrivial. Then inequality (1) has no solutions in D.

Theorem 2. Let $1 + \sigma < q \leq \frac{n(\sigma+1)}{n-4}$, and $\omega(x)$ be nontrivial. Then inequality (2) has no nonnegative solutions in D.

In the case of the Laplace operator, these results were obtained in [1].

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On discrete Hardy type inequality in variable weighted variable Lebesgue spaces

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In this abstract, we establish a two-weight strong inequalities for the multidimensional Hausdorff operator in the scale of different weighted Lebesgue spaces. We also study a similar problem for the dual operator of multidimensional Hausdorff operator. Moreover, we study the problem of compactness of the discrete Hausdorff operator in discrete weighted Lebesgue spaces.

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Approximation by the product means in the generalized Lebesgue spaces

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Let $a = (a_n)$ and $b = (b_n)$ be sequences of nonnegative integers with conditions

$$a_n < b_n \qquad n = 1, 2, 3, ... \quad \text{and} \quad \lim_{n\to\infty} b_n = +\infty.$$

Taking into deferred Cesáro means (D_a^b) given by R. P. Agnew in [1], the product means of D_a^b means with Nörlund (N_p) means are defined by

$$t_n^{D_a^b N_p} = \frac{1}{b_n - a_n} \sum_{k=a_n+1}^{b_n} \left(P_k^{-1} \sum_{v=0}^{k} p_{k-v} s_v\left(f;.\right) \right)$$

where $s_v\left(f;.\right)$ is vth partial sum of Fourier series of a function $f \in L$.

With this perspective, we shall present some results related to trigonometric approximation by the $t_n^{D_a^b N_p}$ means of its Fourier series to functions into $Lip(\alpha, p(x))$ class given in [2].

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On the characteristic features of mathematics in the context of the history of its development

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In the last quarter of 2021, the Belarusian State University (BSU) celebrated its 100th anniversary. An important component of BSU today is the Faculty of Mechanics and Mathematics, which is the successor of the Faculty of Physics and Mathematics established in 1933, later – since 1958 – the Faculty of Mathematics, and received its name in 1975 in connection with the opening of the specialty "mechanics". The study and enhancement of scientific heritage, the preservation of the memory of those scientists who stand at the origins of modern mathematics is an important component of university education.

According to the long-standing traditions of classical university education, the content of teaching students of the Faculty of Mechanics and Mathematics includes a representative list of mathematical disciplines along with disciplines that provide the specifics of future professional activity. Mathematical disciplines are characterized, on the one hand, by a high density of the conceptual and terminological apparatus, on the other hand, by the relative autonomy of the content, which contributes to the "separation" of the perception of concepts included in the program of different mathematical disciplines. In this regard, one of the factors contributing to the productivity of mastering the teaching content is taking into account the characteristic features of mathematics as a science and an educational subject from the standpoint of the historical and genetic approach.

The study of the works of G. H. Hardy (Mathematician's Apology; 1940, J. Hadamard, F. Klein, L. D. Kudryavtsev, as well as R. M. Aslanov [1], O. D. Maksimova, D. M. Smirnov and other authors, as well as their own work experience allowed us to identify the following characteristic features of mathematics as a science and educational subject. That are perfection of the symbolic language of mathematics, abstractness of mathematical objects, evidence-based conclusions, unity of parts of mathematics, universal mathematical constructions, the lack of empirical verification of many mathematical statements [2].

At the same time, the synergetic effects of deepening and concretization of mathematical knowledge on the one hand and increasing abstractness and generality of conclusions on the other, are expressed in the fact that mathematics, firstly, is developing as a science in accordance with the laws of opposition and synthesis of intuition and logic, abstract and concrete, applied and theoretical, constructive and analytical. Secondly, mathematics is increasingly becoming a methodological apparatus for the development of most other sciences and a regulatory tool for analyzing and predicting processes not only in natural science, economics, production, but also in the social and humanitarian sphere. In the conditions of NBIC convergence, convergence and synthesis of achievements of nano-, bio-, information and cognitive technologies and digital transformation of all aspects of society, such characteristics as interdisciplinarity, humanistic orientation, consistency, strengthening of the practice-oriented applied nature of research inevitably rely on the use of methods based on mathematics. This proves that since the second half of the XX century, the listed features of mathematics have been supplemented by such a characteristic as the expansion of the subject of mathematics, which reflects the convergence of mathematics as a method of scientific cognition and the language of science with mathematics as a scientific theory. From the study of spatial forms and quantitative relations of the real world (according to F. Engels) mathematics has moved on to the study of abstract mathematical structures, their properties, relationships and logic of construction.

In this regard, an important component of the training of specialists in mechanical and mathematical specialties at a classical university is the inclusion in the content of teaching facts illustrating the development of ideas, hypotheses and concepts that are the foundations of modern mathematics. From the standpoint of the historical-genetic approach, illustrations of such characteristics of mathematics as logic, evidence and abstract nature of mathematical objects are the problem of infinity (Aristotle in his work "Physics" identified two different types of infinity: potential – an unstoppable process of growth, and actual – a non-finite measure of a real-existing quantity); continuum-hypothesis (Continuum Hypothesis), put forward by G. Kantor later, after substantiating the existence of an infinite number of actual infinities with the help of set theory developed by him; comparison of the ideas of classical and non-classical logics, L. E. J. Brauer's intuitionism (it was supported by Gauss, Kronecker, Poincare, Lebesgue, Borel), B. Hilbert's formalism, G. Frege's logicism; the essence of the unified approach developed by F. Klein to various geometries of Euclidean, affine and projective, Lobachevsky and Riemann geometry using the application of group theory with regard to the allocation of invariants of the corresponding group of transformations. In the Euclidean plane, two points are connected by an invariant to the group of movements, in the similarity group – three points in a common position, in the group of affine transformations – three collinear points, in the group of projective transformations – four collinear points [3].

An illustration of the universal nature and prognosticality of mathematical models can be the theory of conic sections, created in Ancient Greece only 2 thousand years later found application in Kepler's celestial mechanics; the theory of groups developed in connection with the problem of the solvability of algebraic equations in radicals, which almost 60 years after its development (Lagrange, Abel, Galois) was applied for the classification of crystalline substances by the Russian crystallographer E. S. Fedorov; the creation of a binary number system from G.V. Leibniz and D. Buhl algebra to computers and modern computer technology [4].

These facts indicate that another characteristic feature of the development of modern mathematics is the analysis of its foundations, the analysis of the interdependence of its concepts, the structure of individual theories, the analysis of the methods of mathematical proofs and conclusions themselves. Without such an analysis of the fundamentals, the generalizing principles and theories themselves cannot be improved and developed further.

The inclusion of the listed issues of the history and methodology of mathematics development in the content of teaching students of mechanical and mathematical specialties contributes to the development of a motivational and value attitude to learning, bridging the gap between abstract formal and logical constructions of mathematics, the possibilities of computer technology and the tasks of real practice and is the answer of higher education to the scientific and technical challenges of modernity.

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Jackson type inequalities in the Besicovitch-Musielak-Orlicz spaces

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Let f be an arbitrary almost periodic complex-valued Besicovitch function of the class B (B-a.p. function), whose Fourier exponents $\{\lambda_k\}_{k=1}^{\infty}$ have a single limit point at infinity. Let us write its Fourier series in the symmetric form $\sum_{k\in\mathbb{Z}} A_k(f)\mathrm{e}^{\mathrm{i}\lambda_k x}$,where $A_k(f) = \lim_{T\to\infty} \frac{1}{T}\int\limits_0^T f(x)\mathrm{e}^{-\mathrm{i}\lambda_k x}\mathrm{d}x$, $k\in\mathbb{Z}$, $\lambda_0 := 0$, $\lambda_{k+1} > \lambda_k > 0$, $\lambda_{-k} = -\lambda_k$, $|A_k(f)| + |A_{-k}(f)| > 0$, $k > 0$.

Let $\mathbf{M} = \{M_k(t)\}_{k\in\mathbb{Z}}$, $t \ge 0$, be a sequence of nondecreasing convex functions, $M_k(0) = 0$, $M_k(t) \to \infty$ as $t \to \infty$. The modular space (or Besicovitch-Museilak-Orlicz space) $B\mathcal{S}_{\mathbf{M}}$ is the space of $f \in B$-a.p. functions such that

$$\|f\|_{\mathbf{M}} := \sup\Big\{\sum_{k\in\mathbb{Z}} \gamma_k|A_k(f)| : \quad \gamma_k \ge 0,\ \sum_{k\in\mathbb{Z}} M_k(\gamma_k) \le 1\Big\} < \infty.$$

Classical modulus of smoothness of $f \in B\mathcal{S}_{\mathbf{M}}$ of the order $m \in \mathbb{N}$ is defined by

$$\omega_m(f,\delta)_{\mathbf{M}} := \sup_{|h|\le\delta} \|\Delta_h^m(f)\|_{\mathbf{M}} = \sup_{|h|\le\delta} \Big\|\sum_{j=0}^{m}(-1)^j\binom{m}{j} f(\cdot - jh)\Big\|_{\mathbf{M}}.$$

By G_{λ_n} we denote the set of all B-a.p. functions whose Fourier exponents belong to the interval $(-\lambda_n, \lambda_n)$ and $E_{\lambda_n}(f)_{\mathbf{M}} := \inf\limits_{g\in G_{\lambda_n}} \|f - g\|_{\mathbf{M}}$. **Theorem 1.** *For arbitrary numbers $n, m \in \mathbb{N}$ and for any $f \in B\mathcal{S}_{\mathbf{M}}$*

$$E_{\lambda_n}(f)_{\mathbf{M}} \le \frac{1}{2^{\frac{m}{2}} I_n(\frac{m}{2})} \int\limits_0^{\pi} \omega_m\Big(f, \frac{u}{\lambda_n}\Big)_{\mathbf{M}} \sin u\, \mathrm{d}u, \tag{1}$$

where $I_n(\frac{m}{2}) = \inf_{k\in\mathbb{N}, k\ge n} \int_0^{\pi} (1 - \cos\frac{\lambda_k u}{\lambda_n})^{\frac{m}{2}} \sin u\, \mathrm{d}u$. If, in addition $\frac{m}{2} \in \mathbb{N}$, then $I_n(\frac{m}{2}) = \frac{2^{\frac{m}{2}+1}}{\frac{m}{2}+1}$, and the inequality (1) cannot be improved for any $n \in \mathbb{N}$.

Asymptotic stability of a generalized linear difference system with multiple delays

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We consider a linear delay difference system including N delays

$$x_{n+1} - ax_n + B\sum_{i=1}^{N} x_{n-k_i} = 0, \quad n \in \mathbb{N} = \{0, 1, 2, ...\}, \tag{1}$$

where B is a $m \times m$ constant matrix, $a \in [-1, 1] - \{0\}$ is a real number and $k_i (i = 1, 2, ..., N)$ are positive integers satisfying the conditions $k_1 \leq k_2 \leq ... \leq k_N$.

The aim of this paper is to establish necessary and sufficient contitions for (1) to be asymptotically stable, which are described of the matrix B and the delays $\{k_i\}$. Our results extend ones cited in [1].

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To the spectral theory of multiparameter system of operators

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Consider of the multiparameter system

$$(A_{i,0} + \lambda_1 A_{i,1} + \cdots + \lambda_n A_{i,n})\, x_i = 0, \tag{1}$$

where $A_{i,k}$ are bounded operators, acting in the separable Hilbert space $H_i (i = 1, 2, ..., n), \lambda = (\lambda_1, ..., \lambda_n) \in C_n, H = H_1 \otimes \cdots \otimes H_n$.

Definition 1. Let such nonzero element $x_i \in H_i$ exist, that (1) is satisfied. Then element $x_1 \otimes \cdots \otimes x_n$ is name by eigenvector of (1) and $\lambda = (\lambda_1, ..., \lambda_n)$ is corresponding eigenvalue.

Definition 2. Let be $m_1, m_2, ..., m_n$ natural numbers. Element $z_{m_1,m_2,...,m_n} \in H$ is named by $(m_1, m_2, ..., m_n)$-th associated vectors to eigenvector $z_{0,0,...,0}$ of system (1), corresponding to eigenvalue λ_0, if

$$(z_{i_1,i_2,...,i_n}) \subset H_1 \otimes \cdots \otimes H_n, 0 \le i_k \le m_k, k = 1, 2, ..., n$$

and

$$A_k^+(\lambda_0) z_{i_1,i_2,...,i_n} + A_{k,1}^+ z_{i_1-1,i_2,...,i_n} + ... + A_{k,n}^+ z_{i_1,...,i_{n-1},i_n-1} = 0, k = 1, 2, ..., n$$

are true. Δ_i $(i = 1, 2, ..., n)-$ abstract analogue of determinant of (1) and all operators Δ_i act in space H.

Theorem. *Let the following conditions are satisfied:*

a) $KerA_{i,n} = \{\theta\}, Ker\Delta_0 = \{\theta\};$

b)eigenvectors of operator $\Gamma_s = \Delta_0^{-1}\Delta_s, (1 \le s \le n)$ *form the basis in* H.

Then the system of eigen and associated vectors without of the associated vector in direction λ_s *form the basis in* H.

(s-th index at associated vectors is equal to zero).

On a single space with a dominating mixed derivative

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Let E be a Banach space, $r = (r_1, ..., r_n)$, $r_j \geq 0$, e_r-carrier of the vector r. We consider the $SB^r_{p,q,\varphi}(R^n : E)$ space of functions $f \in L_p(R^n : E)$, which has derivatives D^{k^e} of order k^e for all $e \subset e_r$ and for them has the relation

$$\sum_{e^1+e^2=e} \Big\{ \prod_{j\in e^r} \delta_j \Big[\int \cdots \int_{0\leq t_j\leq \delta_j, j\in e^1} \cdots \int_{\delta_j\leq t_j\leq 2, j\in e^2} \bigcap_{j\in e^1} t^{-q(r_j-k_j)-1} \bigcap_{j\in e^2} t^{-q(r_j-k_j+1)-1}$$

$$\Omega^{m^e}\Big(D^{k^e} f : t^e\Big)_{L_p(R^n:E)}\Big]^q dt_1...dt_n\Big\}^{\frac{1}{q}} \leq M^{r^e}_{p,q} \prod_{j\in e}^{n} \varphi_j(\delta^{e^j})$$

where the $\sum\limits_{e^1+e^2=e}$ sum is extended to all possible subsets $e^1, e^2 \subset e \subset e^r$ for which $e^1 \cap e^2 = O$, $e^1 + e^2 = e$ for $\forall e \subset e_r$, $\varphi_i(\delta_j)$ is a continuous non-negative function $\varphi(\delta_j) = o(\delta_j)$, $m = (m_1, ..., m_n)$, $r = (r_1, ..., r_n)$, $k = (k_1, ..., k_n)$, $m_j > r_j - k_j$, $j = 1, ..., n$. $SB^r_{p,q}(R^n : E)$ is a complete normed space. A constructive characterization of space in the form of a convergent series of E valued entire functions of degree 2^{k_j} by x_j and some embedding theorems are established.

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Multiscale models, temporal splitting, and machine learning

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Multiscale problems arise in many applications, and they are typically described by some nonlinear partial differential equations. In many applications, these problems have multiscale nature and contain multiple scales and high contrast. Examples include nonlinear porous media flows (Richards' equations, Forchheimer flow and so on, where the media properties contain many spatial scales and high contrast. Due to high contrast in the media properties, these processes also occur on multiple time scales. E.g., for nonlinear diffusion, the flow can be fast in high conductivity regions and slow in low conductivity regions. Due to a disparity of time scales, special temporal discretizations are often sought, which is the main goal in the context of multiscale problems.

When the media properties are high, the flow and transport become fast and require small time steps to resolve the dynamics. Implicit discretization can be used to handle fast dynamics; however, this requires solving large-scale nonlinear systems. For nonlinear problems, explicit methods are used when possible to avoid solving nonlinear systems. The main drawback of explicit methods is that they require small time steps that scale as the fine mesh and depend on physical parameters, e.g., the contrast. To alleviate this issue, we propose a novel nonlinear splitting algorithm following our earlier works. The main idea of our approaches is to use multiscale methods on a coarse spatial grid such that the time step scales with the coarse mesh size.

Next, we give a brief overview of multiscale methods for spatial discretizations. Multiscale spatial algorithms have been extensively studied for linear and nonlinear problems. For linear problems, many multiscale methods have been developed. These include homogenization-based approaches, multiscale finite element methods, generalized multiscale finite element methods (GMsFEM), constraint energy minimizing GMsFEM (CEM-GMsFEM), nonlocal multi-continua (NLMC) approaches and so on. For high-contrast problems, approaches such as GMsFEM and NLMC have been developed. For example, in the GMsFEM, multiple basis functions or continua were designed to capture the multiscale features due to high contrast. These approaches require careful designing of multiscale dominant modes. For nonlinear problems, linear multiscale basis functions can be replaced by nonlinear maps.

In our works, we design splitting algorithms for solving flow and wave equations. In both cases, the solution space was divided into two parts, the coarse-grid part and the correction part. The coarse-grid solution is computed using multiscale basis functions with CEM-GMsFEM. The correction part uses special spaces in the complement space (the complement to the coarse space). A careful choice of these spaces guarantees that the method is stable. Our analysis in shows that for the stability, the correction space should be free of contrast, and thus, this requires a special multiscale space construction. We extend the linear concepts to nonlinear problems. Splitting algorithms for nonlinear problems have often been used in the literature. Our goal is to use splitting concepts and treat implicitly and explicitly some parts of the solution. As a result, we can use larger time steps that scale with the coarse mesh size.

We make several observations. - Additional degrees of freedom are needed for dynamic problems, in general, to handle missing information.

- We note that restrictive time steps scale with the coarse mesh size, and thus, are much coarser.

We would like to note that the proposed concepts of partially explicit methods can be used in conjunction with other multiscale methods that deal with high contrast. However, multiscale approaches that do not explicitly take into account the high contrast via additional degrees of freedom (when necessary) may not benefit from our proposed concepts. Due to high contrast, small time scales appear, and our goal is to find out those few degrees of freedom that are responsible for the fast dynamics and handle them separately. This condition is rigorously derived and any approaches that split spaces accordingly can be used in the framework of partially explicit methods.

Existence of solution of nonlinear bridge problem with time-varying delay

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This thesis seeks to analyse the mathematical model of the suspension bridge problem with time-varying delay. We consider the following mathematical model for the oscillations of the bridge with strong delay

$$\begin{cases} u_{tt}(x,t) + u_{xxxx}(x,t) + [u-v]_+ + \lambda_1 u_t(x,t) + \\ +\lambda_2 u_t(x, t-\tau_1(t)) = h_1(t,x), \\ v_{tt}(x,t) - v_{xx}(x,t) - [u-v]_+ + \mu_1 v_t(x,t) + \\ +\mu_2 v_t(x, t-\tau_2(t)) = h_2(t,x) \end{cases} \qquad (1)$$

where $0 \le x \le l$, $t > 0$, $u(x,t)$ is state function of the road bed and $v(x,t)$ is that of the main cable; $\tau_1(t)$, $\tau_2(t) > 0$ represents the time-varying delay, λ_1, λ_2, μ_1, μ_2 are real numbers, $[a]_+ = \max\{a, 0\}$.

Let's define the following initial and boundary conditions for the system (1).

$$\begin{cases} u(0,t) = u_{xx}(0,t) = u(l,t) = u_{xx}(l,t) = v(0,t) = v(l,t) = 0, \ t > 0, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad x \in (0,l), \\ u_t(x, t-\tau_1(t)) = f_{01}(x, t-\tau_1(t)), \quad x \in (0,l), t \in (0, \tau_1(0)), \\ v(x,0) = v_0(x), v'(x,0) = v_1(x), x \in (0,l), \\ v_t(x, t-\tau_2(t)) = f_{02}(x, t-\tau_2(t)), \quad x \in (0,l), \ t \in (0, \tau_2(0)). \end{cases} \qquad (2)$$

The existence and uniqueness of the solution is shown by modelling this problem as the Cauchy problem for an operator coefficient equation in a certain space. We also prove the theorem on the existence and uniqueness of the solution of considered problem of homogeneous system.

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Innovative model of planetary reducer

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In order to obtain simple planetary mechanisms from satellite mechanisms, they are usually fastened with 3 central wheels (Fig. 1). In the motor-reducer, one wheel is attached to the rotor of the electric motor, and the drive is attached to the drive shaft, which is taken out of the device.

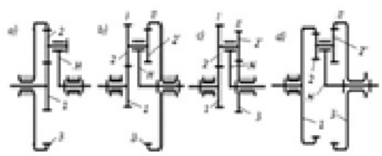

Fig. 1

The design disadvantages of planetary gearboxes are the location of the console on the input shaft of the 1st wheel and the center H of the leading point H, which creates the main central output, torsional deformation and oscillations in the input shaft. Objective: In the innovative device, the central gear is on the central axis base of the leading point.

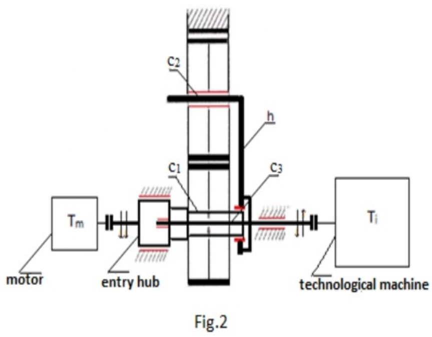

Fig.2

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Let's define Lagrange $L = E - V = \frac{1}{2}J_m\omega_m^2 + \frac{1}{2}J_i\omega_t^2 - \frac{1}{2}c_1(\varphi_g - \varphi_h)^2 - \frac{1}{2}c_2(\varphi_g - \varphi_h)^2 - \frac{1}{2}c_3(\varphi_h - \varphi_i)^2$. Let's look at a special case. Suppose that the power that the engine can generate is large, so we can take $\omega_{\ m} = \text{const}$. Then, assuming $\varphi_{\ m} = \omega_{\ mt}$, Tm and Ti determine the moment of force that the engine must generate. $\frac{c_1 \cdot c_2 \cdot c_3 \cdot u_{ga} \cdot u_{gh}}{c_1 \cdot c_3 + c_1 \cdot c_2 \cdot \omega_{gh} + c_2 \cdot c_3 \cdot \omega_{gh^2}} = C_g$; $\frac{C}{J_1} = k^2\ C_g$ is the coefficient of rigidity of the system. k the specific oscillation frequency of the system. If viscous friction is also taken into account, it is a moment directly proportional to its relative velocity

$T_s = -\beta\frac{d\varphi}{dt}\ \frac{T_s}{J_1} = -\frac{\beta}{J_1}\cdot\frac{d\varphi}{dt}$; $\frac{\beta}{J_1} = 2n$ is indicated by, two cases are considered here: case I $n^2 < k^2$; case II $n^2 > k^2$. The ratio of the amplitude of the forced oscillations to the static deformation is called the dynamic coefficient:

$$k_d = \frac{D}{h/k^2} = \left[\sqrt{\left(\frac{p^2}{k^2} - 1\right)^2 + \frac{4n^2 \cdot p^2}{k^4}}\right]^{-1}$$

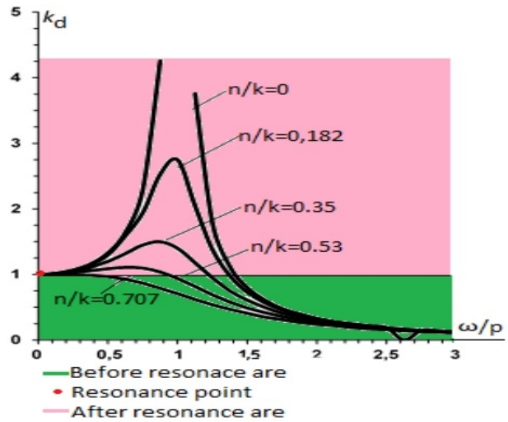

Fig.3

Figure 3 shows diagrams of k_d angle frequency dependence ω/p for several values of n/k. Points $p/k > 1$ are before resonance, and points $p/k > 1$ are after resonance. The diagram shows that after resonance, the dynamic coefficient approaches zero as the frequency of forced oscillations ωp increases.

- the stiffness of the ball on which the center wheel sits and the shaft on which the leading point sits is reduced ($c1c3$), mechanism and important work coefficient rising.

On the Toeplitz product problems

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We study the following zero Toeplitz product problem in the Bergman space $L2$ a (D) : For two bounded symbols f; g; if the product $TfTg$ is identically zero on the Bergman space $L2$ a (D) then can we claim Tf or Tg is identically zero? We partially solve this problem. We also partially solve the following conjecture of Cuckovic : Let f; g are bounded functions with g harmonic. Then $TfTg = 0$ on $L2$ a (D) has only a trivial solution. We also give a new and short proof of a version of Aleman-Vukotic theorem on the zero product of finitely many Toeplitz operators on the Hardy space. Moreover, we give a solution of Lee's problem about toeplitzness of the product of finitely many Toeplitz operators on the Hardy space. Some other related questions are also discussed.

Longitudinal vibrations of a fractal structure rod

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The last few decades, fractional-order differential equations and Abel type integral equations are considered as useful means for studying hereditary and memory properties and various processes [1, 2]. In some cases, they lead to more adequate models than the models based on the derivatives of whole order. In this paper, we attempt to study the models based on the so-called diffusion-wave fractional heat differential equations. Such equations are obtained when in wave equation or in a filtration (heat, diffusion) equation, the second or first derivative with respect to time is replaced by a fractional derivative with respect to time. These equations were successfully applied for modelling various processes, for example, for researching the behavior of visco-elastic materials, for studying filtration processes in porous medium, where fractal structures are formed, etc.

We consider an equation of motion of an infinite visco-elastic rod of density ρ, executing longitudinal vibrations under the action of external load $f(x,t)$ (per unit volume)

$$\frac{\partial\sigma(x,t)}{\partial x} + f(x,t) = \rho\frac{\partial^2 u(x,t)}{\partial t^2} \tag{1}$$

$$\varepsilon(x,t) = \frac{\partial u(x,t)}{\partial x} \tag{2}$$

Here x is corodinate of the rod point, t is time, σ is stress, ε is strain, u is displacement of material element of the rod. We accept the determining physical between stress and strains in the form [1]:

$$\sigma(x,t) = E\xi^{\alpha} D^{\alpha}\varepsilon(x,t), \qquad \alpha\in(0,1) \tag{3}$$

From (1)-(3) we obtain :

$$\frac{\partial^2 u(x,t)}{\partial t^2} = a^2 D^{\alpha}\,\frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t)/\rho \tag{4}$$

where $a^2 = \ E\xi^{\alpha}/\ \rho$.

The solution of the equation (4) is sought in the domain

$$\Omega = \{(x,t) = 0: \quad t > 0,\ -\infty < x < +\infty\}.$$

Boundary and initial conditions are accepted one obtained in the form:

$$u(-\infty,t) = u(+\infty,t),\ t > 0$$

$$u\left(x,0\right) = \ \frac{\partial u(x,0)}{\partial t} = 0,\ \ -\infty < x < +\infty$$

Analytic expressions for solving the problem are obtained in the form :

$$u\left(x,t\right) = \left(\frac{1}{\rho\pi}\right)\int_{-\infty}^{+\infty}\int_{0}^{+\infty} t^2 E_{2-\alpha,3}\left(-p^2 t^{2-\alpha}\right) cos\left(px\right) dp$$

Here $E_{2-\alpha,3}(-p^2t^{2-\alpha})$ s a Mittag-Leffler function tabulated and known from mathematical literature.

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A mixed problem for hyperbolic equations with irregular boundary conditions

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Problem statement. Find the classical solution of the hyperbolic equation

$$\frac{\partial^4 u}{\partial t^4} - 2a^2 \frac{\partial^4 u}{\partial x^2 \partial t^2} + a^4 \frac{\partial^4 u}{\partial x^4} = F(x,t),\ 0 < x < 1, \quad 0 < t < \infty$$

satisfying the irregular boundary conditions

$$u|_{x=0} = \gamma_1(t),\ u|_{x=1} = \gamma_2(t), \left.\frac{\partial u}{\partial x}\right|_{x=0} = \gamma_3(t),$$

$$\left.\frac{\partial^3 u}{\partial x^3}\right|_{x=1} - \frac{1}{a^2} \left.\frac{\partial^3 u}{\partial x \partial t^2}\right|_{x=1} = \gamma_4(t), t > 0,$$

and initial conditions

$$\left.\frac{\partial^k u}{\partial t^k}\right|_{t=0} = f_k(x), \quad 0 < x < 1, \quad k = \overline{0,3}.$$

Theorem *Let the constraints hold. Then if the problem has a classical solution $u(x,t)$ having a constraint, then for this solution the following integral representation takes place [1].*

$$u(x,t) = \int\limits_{y} \phi(x,t,\lambda) d\lambda.$$

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Theorems about convergence for one boundary value problem second-order differential operator

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Consider the following boundary value problem

$$ly = -y'' + q(x)\,y = \lambda y, x \in G = (0,1) \tag{1}$$

$$\alpha y(0) + \beta y'(0) - y(1) = 0 \tag{2}$$

$$\gamma y(0) + \delta y'(0) - y'(1) = 0 \tag{3}$$

where is $q(x) \in L_1(G) -$ a real function, $\alpha = c_1 e^{i\theta}$, $\beta = c_2 e^{i\theta}$, $\gamma = c_3 e^{i\theta}$, $\delta = c_4 e^{i\theta}$; real numbers, $i = \sqrt{-1}$. Self-adjoint conditions has the form $c_1c_4 - c_2c_3 = 1$.

Let $\{y_k(x)\}_{k=1}^{\infty} -$ be a complete orthonormal system in $L_2(\,G\,)$consisting of eigenfunctions and $\{\lambda_k\}_{k=1}^{\infty}$ – be the correspanding sistem of eigenvalues of boundary value problem (1)-(3).

By $W_p^1(G),\ p \geq 1$, we denote the class of functions $f(x)$, absolutely continuous on the interval $\overline{G}$ for which $f'(x) \in L_p(G)$. Denote $\mu_k = \sqrt{\lambda_k}$, introduce a partial sum of the orthogonal expansion of the function $f(x) \in W_p^1(G), p > 1$, with respect to the system $\{y_k(x)\}_{k=1}^{\infty}$:

$\sigma_\nu(x,f) = \sum\limits_{\mu_k \leq \nu} f_k\, u_k(x)$, $\nu > 0$, where $f_k = (f, u_k) = \int\limits_0^1 f(x)\,\overline{u_k(x)}dx$.

We introduce the difference $R_\nu(x,f) = f(x) - \sigma_\nu(x,f)$.

Theorem 1. *Assume that a function $f(x)$ of the class $W_p^1(G)$, $p > 1$, and $\beta \neq 0$. Then the spectral expansion of the function $f(x)$ with respect to the system $\{y_k(x)\}_{k=1}^{\infty}$ converges absolutely and uniformly on the interval $\overline{G} = [0,1]$ and the following estimate holds for the remainder*

$$\|R_\nu(\cdot,f)\|_{C[0,1]} \leq$$

$$\leq const\left\{C_1(f)\,\nu^{-1} + \nu^{-\beta}\,\|f'\|_p + \nu^{-1}\left(\|f\|_\infty + \|f'\|_p\right)\|q\|_1\right\} \tag{4}$$

where $\nu \geq 1, p^{-1} + q^{-1} = 1,\ \beta = \min\{2^{-1},\ q^{-1}\}, \|\cdot\|_p = \|\cdot\|_{L_p(G)}$; *const is independent of the function* $f(x)$; $C(f) = [|\gamma| + |\alpha|\ |\delta|\ |\beta^{-1}|]\ |f(1)| + [|\beta^{-1}| + |\alpha|\,|\beta^{-1}|]\, f(0)$.

Theorem 2. *Let* $\beta = 0$ *and assume that a function* $f(x)$ *of the class* $f(x) \in W_p^1(G)$, $p > 1$, *satisfies the relation* $f(1) = \alpha f(0)$.*Then the spectral expansion of the function with respect to the system* $\{y_k(x)\}_{k=1}^{\infty}$ *converges absolutely and uniformly on the interval and for the remainder* $R_\nu(x, f)$ *the estimate (4) is true with constant* $C_1(f) = |\gamma f(1)|$.

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First and second order optimality conditions for vector optimization problems

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In the talk we present a unified approach to deriving first and second order local optimality conditions, both necessary and sufficient, for solutions of vector optimization problems with non-solid positive cone.

Let X and Y be Banach spaces over the real field $\mathbb{R}$. The space Y is supposed to be ordered by a strict partial order $\prec$ such that for any y_1, $y_2 \in Y$ one has $y_1 \prec y_2$ if and only if $y_2 - y_1 \in P$, where $P \subset Y$ is an asymmetric convex cone the interior of which is not supposed to be non-empty.

The talk deals with a vector optimization problem

$$\prec\text{–minimize}\, F(x) \text{ subject to } x \in Q, \qquad (VOP)$$

where $F : X \to Y$ is a mapping from X into Y, Q is a subset of X.

A point $x^0 \in Q$ is called *a local $\prec$–minimizer* for (VOP) if there exists a neighborhood $N(x^0)$ of x^0 such that $F(x) \not\prec F(x^0)$ for all $x \in Q \cap N(x^0)$.

We assume that the objective mapping F is differentiable in one or another sense, in particular, we admit that F is twice parabolic directionally differentiable. In the case when the objective mapping F is twice Fréchet differentiable and the feasible set Q is the whole space X the $\prec$–minimality conditions are presented in the prime and dual forms. The first order dual necessary $\prec$–minimality condition has the form of the Lagrange multipliers rule while the second order dual necessary $\prec$–minimality condition asserts that the maximum of the family quadratic forms on the cone of critical vectors is nonnegative.

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The Wiener criterion for the heat equation in terms of the potential

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The regularity of boundary points with respect to the Dirichlet problem for the heat equation and parabolic equations has been studied in various terms ([1]- [4]). In this paper, in terms of a parabolic potential, the Wiener criterion for the regularity of boundary points of a bounded domain with respect to the Dirichlet problem is given.

Let $B \subset R^{n+1}, (t,x) = (t, x_1, \dots, x_n)$ and

$$P_B\left(t,x\right) = \int_B K(t-\tau, x-\xi) d\mu(\tau, y) \tag{1}$$

be heat potential with generated of Weierstrass kernel

$$K\left(t,x\right) = \begin{cases} \quad (4\pi t)^{-\frac{n}{2}} \cdot exp\left\{-\frac{|x|^2}{4t}\right\}, & t > 0 \\ 0, & t \leq 0 \end{cases} \tag{2}$$

here B is a Borel set and μ is a Borel measure.

Denote for $\lambda > 1$ and $m \in N \cup \{0\}$ paraboloids:

$P_m = \left\{(t,x) \mid |x|^2 < -\lambda^m \cdot t, t < 0\right\}$.

Let $B_{m,k} = (P_{m+1} \backslash P_m) \cap \left[-t_k; -\frac{t_k}{2}\right]$, where $t_{k+1} = \frac{t_k}{4}$, $m, \ k \in N \cup \{0\}$, $t_0 > 0$ and denote cylinders

$C_{m,k} = \{(t,x) \mid -t_k < t < 0, \ |x| < a \cdot \rho_{m,k}\}$, where $a > 0$ the chosen absolute constant depending only on the dimension n of the space and denote by $S_{m,k}$ the lateral surface of the cylinder $C_{m,k}$. The measure μ on B called admissible, if

$$P_B\left(t,x\right) = \int_B K\left(t-\tau, x-\xi\right) d\mu\left(\tau, \xi\right) \leq 1 \tag{3}$$

in R^{n+1}. The number $cap\left(B\right) = sup\mu B$,where the supremum is taken by the all possible admissible measures μ is called *thermal capacity* of the set B.

Let's call $T_{m,k} = C_{m,k} \backslash P_m$ trapezoids and denote by $T_{m,k}^{(j)}, j = 1, 2, \ldots, n_0(n)$ corresponding minimal finite partition $T_{m,k}$, for which the following

$$|x - \xi| \leq |\xi|$$

inequality is fulfilled at $(t, x) \in T_{m,k+1}^{(j)}$ and $(\tau, \xi) \in T_{m,k}^{(j)}$ for every fixed $j \in \{1, \ldots, n_0\}$.

Lemma 1. *If* $(t, x) \in T_{m,k+1}^{(j)}$ *and* $(\tau, \xi) \in T_{m,k}^{(j)}$*, then* $\frac{|x-\xi|^2}{t-\tau} \leq \frac{|\xi|^2}{-\tau}$. $(*)$

Lemma 2. *There exist absolute constants $C_1 > 0$ and $C_2 > 0$ depending only on fixed numbers λ, a and n such that holds*

$$\sup_{S_{m,k}} P_{B_{m,k}}\ (t, x) \leq C_1 P_{B_{m,k}}(0, 0) \tag{4}$$

and also such finite partition that for every fixed $j \in \{1, \ldots, n_0\}$

$$\inf_{T_{m,k,+1}^{(j)}} P_{B_{m,k}}(t, x) \geq C_2 P_{B_{m,k}}(0, 0), \tag{5}$$

moreover $C_2 > C_1$.

Lemma 3.(about the growth of positive solutions). *Let bounded domain $D \subset R^{n+1}$ containing in cylinder $C_{m,k}$, intersecting by cylinder $C_{m,k+1}$: $D \cap C_{m,k+1} \neq \emptyset$ and $u(t, x)$ be a solution of the heat equation*

$$Hu\,(t, x) \equiv \Delta u\,(t, x) - u_t(t, x) = 0,$$

positive in D, continuous in $\overline{D}$ and vanishing on such part of the parabolic boundary $\partial_p D$ of the domain D, which lies strongly inside $C_{m,k}$. Then for every $j \in \{1, \ldots, n_0\}$ we have

$$\sup_{D \cap T_{m,k}^{(j)}} u(t, x) \geq \left(1 + \eta \cdot P_{D^c \cap T_{m,k}^{(j)} \cap B_{m,k}}\,(0, 0)\right) \sup_{D^c \cap T_{m,k+1}^{(j)}} u(t\ , x). \tag{6}$$

Our main result the criterion for boundary regularity is the following

Theorem. *A point $(0, 0) \in \partial_p D$ is regular if and only if*

$$\sum_{m,k=1}^{\infty} P_{D^c \cap T_{m,k}^{(j)} \cap B_{m,k}}\,(0, 0) = +\infty. \tag{7}$$

An equivalent condition is

Corollary. *A point* $(0,0) \in \partial_p D$ *is regular if and only if*

$$\sum_{k=1}^{\infty} P_{D^c \cap A\left((0,0),\ e^{-k}\right)}(0,0) = \sum_{k=1}^{\infty} e^{kn}/2cap\left(D^c \cap A\left((0,0)\,,\ e^{-k}\right)\right) + \infty, \qquad (8)$$

where

$$A\left((0,0)\,,\ c\right) = \left\{(t,x):\ (4\pi c)^{-n/2} \leq K\left(t,x\right) \leq (2\pi c)^{-n/2},\ c > 0\right\}.$$

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A sufficient condition for the regularity of boundary points for solutions of second-order parabolic equations with respect to the Dirichlet problem

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For the Dirichlet problem .

$$Lu \equiv \sum_{i,k=1}^{n} a_{ik}(t,x)\, u_{x_i x_k} - u_t(t,x) = 0, \qquad u|_{\partial_p D} = f(t,x) \tag{1}$$

in the bounded domain $D \subset \mathbb{R}^{n+1}$ the question of regularity of boundary points is studied. Matrix $A(t,x) = \{a_{ik}(t,x)\}$ satisfy the condition of uniform ellipticity and symmetry. We accept the following notations: Let $P_m = \{(t,x) \mid |x|^2 \ < \lambda^m \cdot (-t)\ , t < 0\ ,\ \ \lambda > 1\}$,$B_{m,k} = (\overline{P_{m+1} \backslash P_m} \cap \left[-t_k; -\frac{1}{2}t_k\right]$ for $m,k \in N$, and $t_k = \frac{t_{k-1}}{4}$, $t_0 > 0$. $C_{m,k} = (t,x) \bigm| -t_k < t < 0,\ \ |x| < a\rho_{m,k}$, $a > 0, \rho^2_{m,k} = \lambda^m \cdot t_k \}$, $T_{m,k} = C_{m,k} \backslash P_m$. Denote by $T^{(j)}_{m,k}$, $j \in \{1,2,\ldots,n_0(n)\}$, the minimal finite partition of $T_{m,k}$ such that for which the following inequality

$$|x - \xi| \le |\xi| \tag{2}$$

is fulfilled at $(t,x) \in T^{(j)}_{m,k+1}$ and $(\tau,\xi) \epsilon T^{(j)}_{m,k}$.

Lemma 1. *There exist absolute constants $C_1 > 0$ and $C_2 > 0$, depending only on fixed numbers λ, a, s, β such that holds*

$$\sup_{S_{m,k}} P_{H^{(j)}_{m,k}}(t.x) \le C_1 P_{H^{(j)}_{m,k}}(0.0), \tag{3}$$

and also for the finite partition and for all fixed $j \in \{1,\ldots,n_0(n)\}$ such that

$$\inf_{T^{(j)}_{m,k+1}} P_{H^{(j)}_{m,k}}(t,x) \ge C_2\ P_{H^{(j)}_{m,k}}(0,0). \tag{4}$$

moreover $C_2 > C_1$, where

$$H^{(j)}_{m,k} = D^c \bigcap T^{(j)}_{m,k}, P_B(t,x) = \int_B K_{s,\beta}(t-\tau, x-\xi)\, \mu(\tau,\xi)$$

is the parabolic potential, with the generated kernel of

$$K_{s,\beta}(t,x) = \begin{cases} t^{-s} exp\left\{-\frac{|x|^2}{4\beta t}\right\}, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

B is a Borel set and μ is a Borel measure.

Lemma 2. *Let bounded domain $D \subset R^{n+1}$ containing in cylinder $C_{m,k}$, intersecting by cylinder $C_{m,k+1}$: $D \cap C_{m,k+1} \neq \emptyset$ and $u(t,x)$ be a solution of the parabolic equation*

$$\sum_{i,k=1}^{n} a_{ik}(t,x)\, u_{x_i x_k} - u_t(t,x) = 0,$$

positive in D, continuous in $\overline{D}$ and vanishing on such part of the parabolic boundary $\partial_p D$ of the domain D, which lies strongly inside $C_{m,k}$. Then for every $j \in \{1, \ldots, n_0\}$ we have

$$\sup_{D \cap T^{(j)}_{m,k}} u(t,x) \geq \left(1 + \eta \cdot P_{H^{(j)}_{m,k}}(0,0)\right) \cdot \sup_{D^c \cap T^{(j)}_{m,k+1}} u(t,x)$$

Main result. Theorem. *In order to the boundary point$(0, 0 \in \partial_p D)$ to be regular with respect to the Dirichlet problem (1) sufficiency is that*

$$\sum_{m,k=1}^{\infty} P_{H^{(j)}_{m,k}}(0,0) = +\infty. \qquad (6).$$

The bilateral estimates of parabolic potentials with polar value in special domains

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At studying the qualitative properties of solutions of parabolic equations, it is important to estimate the parabolic potentials generated by Weierstrass-type kernel in special domains [1].

Let

$$P_B(t,x) = \int_B K_{S,\beta}(t-\tau,\, x-\xi)\, d\mu\ (\tau,\xi)$$

be parabolic potential, with the generated kernel of Weierstrass type

$$K_{s,\beta}(t,x) = \begin{cases} t^{-s} \cdot e^{-\frac{|x|^2}{4\beta t}}, & t>0 \\ 0, & t \le 0 \end{cases}$$

here s and β are positive numbers, B is a Borel set, and μ is a Borel measure.

Denote for $\lambda > 1$ and $m \in N \cup \{0\}$ the paraboloids:

$$P_m = \left\{(t,x)\ :\ |x|^2 < -\lambda^{\mathrm{m}} \cdot t,\ t<0\right\}.$$

Let

$$B_{m,k} = (P_{m+1} \backslash P_m) \bigcap \left[-t_k; -\frac{1}{2}t_k\right].$$

where $t_{k+1} = \frac{t_k}{4},\ k \in N,\ t_1 > 0,\ C_{m,k} = \{(t,x) : -t_k < t < 0,\ |x| < a \cdot \rho_{m,k}\}$, where $a > 0$ and $\rho_{m,k}^2 = \lambda^m \cdot t_k$ and denote by $S_{m,k}$ the lateral surface of the cylinder $C_{m,k}$.

The measure μ on B is called admissible, if

$$P_B(t,x) = \int_B K_{S,\beta}(t-\tau,\, x-\xi)\, d\mu\ (\tau,\xi) \le 1$$

in R^{n+1}. The number

$$cap_{s,\beta}(B) = \sup\ \mu B,$$

where the supremum is taken by the all possible admissible measures μ is called parabolic (s, β) capacity of the set B.

Let $T_{m,k} = C_{m,k} \backslash P_m$, and denote by $T_{m,k}^{(j)}$, $j = 1, 2, ..., n_0(n) \quad = 2^{n+1}$ the minimal finite partition of $T_{m,k}$, for which the following

$$|x - y| \leq |y| \tag{$*$}$$

is fulfilled, at $(x, t) \in T_{m,k+1}^{(j)}$ and $(y, \tau) \in T_{m,k}^{(j)}$.

Lemma. *If* $(x, t) \in T_{m,k+1}^{(j)}$ *and* $(y, \tau) \in T_{m,k}^{(j)}$ *, then*

$$\frac{|x - y|^2}{t - \tau} \leq \frac{|y|^2}{-\tau}. \tag{$**$}$$

Theorem. *There exist the following absolute constants* $C_1 > 0$ *and* $C_2 > 0$, *depending only on fixed numbers* λ, a, s *and* β, *such that holds*

$$\sup_{S_{m,k}} P_{B_{m,k}}(t, x) \leq C_1 \cdot P_{B_{m,k}}(0, 0), \tag{1}$$

and also such finite partition that at some j

$$\inf_{T_{m,k+1}^{(j)}} P_{B_{m,k}}(t, x) \geq C_2 \cdot P_{B_{m,k}}(0, 0), \tag{2}$$

moreover $C_2 > C_1$.

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Approximation by deferred Cesáro-Matrix product means

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Let $T = (u_{j,k})$ be an infinite triangular matrix satisfying the Silverman-Toeplitz conditions. The deferred Cesáro-Matrix product mean is defined by

$$t_n^{DT}(f;x) := \frac{1}{b_n - a_n} \sum_{m=a_n+1}^{b_n} \sum_{k=0}^{m} u_{m,k} S_k(f;x),$$

where $a = (a_n)$ and $b = (b_n)$ are sequences of nonnegative integers with conditions $a_n < b_n$, $n = 1, 2, 3, ...$ and $\lim_{n\to\infty} b_n = +\infty$, and $S_n(f;x)$ denotes n^{th} partial sum of Fourier series of $f \in L$. Also, we say that $f \in W(L^p, \xi(t))$ if the condition

$$\|(f(x+t) - f(x))sin^{\beta}(x/2)\|_p = O(\xi(t))$$

holds, where $\xi(t)$ is a positive increasing function and $p \geq 1, \beta \geq 0$.

In this study we obtain the error estimates of approximation to conjugate of a function f (2π-periodic) in the weighted generalized Lipschitz class $W(L^p, \xi(t))$, $p \geq 1$, by the product mean of its conjugate Fourier series [1].

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On an analogue of Bernstein-Walsh type estimations for the derivative of algebraic polynomials

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Let $G \subset \mathbb{C}$ be a bounded Jordan region, $L := \partial G$; $P_n(z)$ - arbitrary algebraic polynomial, $\deg P_n \leq n$, $n \in \mathbb{N}$, and $h(z)$ be a generalized Jacobi weight function, defined as follows:

$$h(z) := \prod_{j=1}^{l} |z - z_j|^{\gamma_j}, \ \gamma_j > -2, \ j = 1, 2, ..., l, \ z \in \mathbb{C}.$$

Let $0 < p \leq \infty$. For the Jordan region G, we introduce:

$$\begin{aligned} \|P_n\|_p \ &: = \|P_n\|_{A_p(h,G)} := \left(\iint\limits_G h(z) |P_n(z)|^p d\sigma_z \right)^{1/p}, 0 < p < \infty, \\ \|P_n\|_\infty \ &: = \|P_n\|_{A_\infty(1,G)} := \max_{z \in \overline{G}} |P_n(z)|, \ p = \infty; \\ A_p(1, G) \ &\equiv \ A_p(G), \end{aligned}$$

where σ be the two-dimensional Lebesgue measure.

In this paper, we study the problem on pointwise estimates of the derivatives $\left|P_n^{(m)}(z)\right|$ for $m \geq 1$ in unbounded regions of the complex plane as the following type:

$$\left|P_n^{(m)}(z)\right| \leq \eta_n \|P_n\|_p, z \in \mathbb{C} \setminus \overline{G},$$

where $\eta_n := \eta_n(G, h, p, z) \to \infty$ as $n \to \infty$, depending on the properties of G, h.

g-Hölder continuity of Riesz potential on homogeneous spaces

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Let $d > 0$ and $0 \leq \theta \leq 1$. A spaces of homogenous type $(X, \rho, \mu)_{d,\theta}$ is a set X together with a quasi-metric ρ and a nonnegative Borel measure μ on X with $supp\ \mu = X$, $dimX = \infty$ and there exists a constant $c > 0$ such that for all $r > 0$ and all $x, y, z \in X$

$$Cr^d < \ \mu(B(x,r)) < Cr^d$$

and

$$|\rho\,(x,y) - \rho(z,y)| < C\rho(x,z)^{\theta}[\rho\,(x,y) + \rho(z,y)]^{1-\theta}$$

Consider the Riesz potential

$$I_\beta f\,(x) = \int_X \rho\,(x,y)^{\beta-d} f\,(y)\, d\mu(y)$$

For the finiteness of the Riesz potential μ-almost everywhere on X it is necessary a sufficient that

$$\int_X (1 + \rho\,(x_0, y))^{\beta-d} f\,(y)\, d\mu\,(y) < \infty. \tag{1}$$

where x_0 is any fixed point on X.

Take a function

$$g\,(r) = \left(\int_0^r \frac{1}{tln^{\frac{\sigma}{p-1}}(1+\frac{1}{t})} dt\right)^{1-\frac{1}{p}}$$

where $\sigma > p - 1 > 0$.

Theorem. *Let be given space $(X, \rho, \mu)_{d,\theta}$ and $p = \frac{d}{\beta} > 1$. If f satisfies (1) and*

$$\int_X |f(x)|^p (ln\,(1 + f\,(y)))^p d\mu(y) < \infty$$

then

$$\lim_{\beta(x,y)\to 0} \frac{|I_\beta f\,(x) - I_\beta f\,(y)|}{g(\rho\,(x,y))} = 0.$$

Feedback control of motion speeds of lumped sources during plate Heating

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We investigate the problem of synthesize the optimal control of moving lumped heat sources for heating a thin plate. The trajectories of heat sources and the speeds of their motion described by ordinary differential equations are optimized. The current values of the speeds of movement of the sources are determined depending on the temperature at the points of measurement, the location of which is being optimized. We proposes to use the linear dependence of the control actions by the motion of the sources on the measured temperature values. Constant coefficients involved in these dependencies are the desired feedback parameters. The formulas for the gradient components of the objective functional allowing for the numerical solution of the problem using of the first-order optimization methods are obtained.

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Boundedness of the discrete Ahlfors-Beurling transform on discrete Morrey Spaces

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Let $h = \{h_n\}_{n\in Z_C} \in l_p\,,\ p \geq 1$. Namely, the sequence $\tilde{B}(h) = \left\{ \left(\tilde{B}h\right)_n \right\}_{n\in Z_C}$ is called the Ahlfors-Beurling transform of the sequence h, where

$$\left(\tilde{B}h\right)_n = \sum_{m\in Z_C,\ m\neq n} \frac{h_m}{(n-m)^2}\ ,\quad n\in Z_C.$$

The classical Morrey spaces $M_{\lambda,p},\ 0 \leq \lambda \leq \frac{1}{p},\ 1 \leq p \leq \infty$, consist of the functions $f \in L_{p,loc}$ for which the following norm is finite

$$\|f\|_{M_{\lambda,p}} = \sup_z \sup_{r>0} \left[|B(z,r)|^{-\lambda} \|f\|_{L_p(B(z,r))} \right].$$

We note that if $\lambda = 0$, then $M_{\lambda,p} = L_p$;if $\lambda = \frac{1}{p}$, then $M_{\lambda,p} = L_\infty$. In case, $p > 1,\ 0 \leq \lambda \leq \frac{1}{p}$, F. Chiarenza and M. Frasca [1] showed the boundedness of the Hardy-Littlewood maximal operator, the fractional integral operator and a singular integral operator in the Morrey spaces.

Hence, in particular, it implies the boundedness of the Ahlfors-Beurling transform in Morrey spaces. It means that, in case $p > 1,\ 0 \leq \lambda \leq \frac{1}{p}$, for any $f \in M_{\lambda,p}$, we have $Bf \in M_{\lambda,p}$, and there exists $C_{\lambda,p} > 0$such that

$$\|Bf\|_{M_{\lambda,p}} \leq C_{\lambda,p} \cdot \|f\|_{M_{\lambda,p}}$$

holds for all $f \in M_{\lambda,p}$.

In [2], the authors introduced a discrete analogue of Morrey spaces and studied their inclusion properties. For $m \in Z_C$ and $n \in N \bigcup \{0\}$ define $S_{m,n} = \{ k \in Z_C\ :\ \|k-m\| \leq n \}$. Following standard conventions, we denote the cardinality of a set S by $|S|$. Then we have $|\,S_{m,n}| = (2n+1)^2$ for all $m \in Z$ and each $n \in N \bigcup \{0\}$. Discrete Morrey spaces $m_{\lambda,p}\,,\ 0 \leq \lambda \leq \frac{1}{p}\,,\ 1 \leq p < \infty$, consist of the sequences $h = \{h_n\}_{n\in Z_C}$ for which the following norm is finite

$$\|h\|_{m_{\lambda,p}} \sup_{m\in Z_C} \sup_{n\in N \bigcup\{0\}} \left[|S_{m,n}|^{-\lambda} \left(\sum_{k\in S_{m,n}} |h_k|^p \right)^{1/p} \right].$$

Theorem: *Let* $1 \leq p < \infty\,,\ \ 0 \leq \lambda \leq 1/p$. *For any* $h \in m_{\lambda,p}$ *we have* $\tilde{B}(h) \in m_{\lambda,p}$, *and there exists* $c_{\lambda,p} > 0$ *such that*

$$\left\|\tilde{B}(h)\right\|_{m_{\lambda,p}} \leq c_{\lambda,p} \cdot \|h\|_{m_{\lambda,p}}$$

holds for all $h \in m_{\lambda,p}$.

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On a priori estimation of Hölder norm of solutions of a degenerate elliptic equation by $p(x)-$ Laplacian.

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In the domain $D \subset R^n$, $n \geq 2$, we consider a family of elliptic equations

$$L_\varepsilon u = div(\omega_\varepsilon(x)a(x)|\nabla u|^{p(x)-2}\nabla u) = 0 \qquad (1)$$

with positive weight $\omega_\varepsilon(x)$ and exponent $p(x)$. Here $a(x) = \{a_{ij}(x)\}$ is a real symmetric matrix with measurable elements. It is assumed that the following conditions are fulfilled with respect to the coefficients of the operator L

$$\mu\,|\xi|^2 \leq \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \leq \mu^{-1}\,|\xi|^2\,, \mu \in (0,1]\,. \qquad (2)$$

It is supposed that the domain D is divided by the hyperplane $\Sigma = \{x : x_n = 0\}$ into the parts $D^{(1)} = D\bigcap\{x : x_n > 0\}$, $D^{(2)} = D\bigcap\{x : x_n < 0\}$ and

$$\omega_\varepsilon(x) = \begin{cases} \varepsilon,\ if\ \ x \in D^{(1)} \\ 1,\ if\ \ x \in D^{(2)}, \end{cases} \qquad \varepsilon \in (0,1], \qquad (3)$$

$$p(x) = \begin{cases} q,\ if\ x \in D^{(1)} \\ p,\ if\ x \in D^{(2)}, \end{cases} \qquad 1 < q < p. \qquad (4)$$

Under the solution of equation (1) we understand the function $u \in W_{loc}(D)$ satisfying the integral identity

$$\int_D \omega_\varepsilon(x)a(x)|\nabla u|^{p(x)-2}\nabla u \cdot \nabla\varphi dx = 0 \qquad (5)$$

on the trial functions $\varphi \in C_0^\infty(D)$. For the exponent $p(\cdot)$ given by the equality (4), the smooth functions are dense in $W_{loc}(D)$ (see. [1]), and as a result in the integral identy (5) as trial functions we can take finite functions from $W_{loc}(D)$.

Theorem. There exists such a constant $\alpha \in (0,1)$ dependent on the dimension of the space n and constants μ, p, q from the condition (3), that the family $\{u^\varepsilon(x)\}$ is compact in $C^\alpha(D')$ in arbitrary subdomain $\bar{D}' \subset D$.

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Completeness of elementary solutions of the initial-boundary value problem for parabolic equations

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Consider the initial-boundary value problem for parabolic equation

$$\frac{\partial u(x,t)}{\partial t}+a(x)\frac{\partial^{2m}u(x,t)}{\partial x^{2m}}+\sum_{k=0}^{2m-1}a_k(x)\frac{\partial^k u(x,t)}{\partial x^k}=0 \tag{1}$$

$$L_v u=\alpha_v^{(m_v)}(0,t)+\beta_v u^{(m_v)}(1,t)+\sum_{p=1}^{N_v}\delta_{vp}u_x^{(m_v)}(x_{vp},t)+T_v u(t)=0 \tag{2}$$

$$v=1\div 2m,\;\; u(x,0)=\varphi(x) \tag{3}$$

where $x_{vp}\in(0,1), m_v\le 2m$.

The spectral problem corresponding to problem (1),(2) has the following form:

$$\lambda(x)+a(x)u^{(2m)}(x)+\sum_{k=0}^{2m-1}a_k(x)u(x)=0 \tag{4}$$

$$L_v u=0,\;\; v=1\div 2m \tag{5}$$

Functions of the form

$$u_j(x,t)=e^{\lambda_j t}\left(\frac{t^{k_j}}{k_j!}u_{j0}(x)+\frac{t^{k_j}}{k_j-1}u_{j1},...,u_{jk_j}(x)\right) \tag{6}$$

is an elementary solution of the problem (1)-(3) if and only if $u_{j0}(x)$, $u_{j1}(x),...,u_{kj}(x)$ is a chain of root functions of problem (4),(5) corresponding to the eigenvalue λ_j .

Under certain conditions, it is proved that problem (1)-(3) has a unique solution and there are numbers c_{jn} such that

$$\lim_{n\to\infty}\max_{t\in[0,T]}\left(\left\|u_t'(x,t)-\sum_{j=1}^{n}c_{jn}u_{jt}'(x,t)\right\|_{L_2(0,1)}+\left\|u(x,t)-\sum_{j=1}^{n}c_{jn}u_j(x,t)\right\|_{W_2^{2m}(0,1)}\right)=0$$

where $u(x,t)$ is a solution to problem (1)-(3), and $u_j(x,t)$ are elementary solutions to problem (1)-(3).

Obtaining the fractional order differential equation for generating function of the boundary functional of the semi-Markov walk process

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Let there be given a sequence of independent and identically distributed pairs of random variables $\{\xi_k, \zeta_k\}_{k\geq 1}$ defined on the probability space (Ω, F, P), where the random variables ξ_k and ζ_k, $k=\overline{1,\infty}$ are positive and independent. We construct the following semi-Markov random walk process:

$$X\left(t\right) = z + t - \sum_{i=0}^{k-1}\zeta_i, \qquad \sum_{i=0}^{k-1}\xi_i \leq t < \sum_{i=0}^{k}\xi_i \quad k=\overline{1,\infty},$$

where $\xi_0 = \zeta_0 = 0$.

Let us define the random variable:

$$\nu_1^0 = \min\left\{k : z + \sum_{i=1}^{k}(\xi_i - \zeta_i) \leq 0\right\}.$$

The main purpose of this work is to find generating function of the conditional distribution ν_1^0.

We set

$$\psi(\,u\,|\,z) = E\left(u^{\nu_1^0}\middle| X(0)=z\right) = \sum_{k=1}^{\infty} u^k P\left\{\nu_1^0 = k\,|X(0)=z\right\}, \quad 0<u\leq 1,$$

$$Q(u\,|z) = e^{-\beta z}\psi(u\,|z).$$

In the work, a fractional order differential equation with constant coefficients is obtained:

$$D_z^{\alpha+1}(Q\left(u\,|z\right)) + (\mu+\beta)D_z^{\alpha}(Q\left(u\,|z\right)) - \mu\beta^{\alpha}uQ\left(u\,|z\right) = 0.$$

On limit integral equations of Fredholm type in the space of bounded functions

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In the article [1] a new integral equations of Fredholm type in the spaces $\boldsymbol{C}B\,(\mathbb{R})$ of bounded continuous functions in $\mathbb{R}$ were introduced:

$$\varphi(x) = f(x) + \lambda \lim_{T\to\infty} \frac{1}{T} \int_0^T K(x,\xi)\varphi(\xi)d\xi. \tag{1}$$

The analogs of Fredholm functions are defined also. It was proven the analog of first theorem of Fredholm.

Theorem 1. Let λ be a real number such that $D(\lambda) \neq 0$ and $f(x) \in \boldsymbol{CB}\,(\mathbf{R})$. Then there exists a sequence μ of natural numbers such that the equation (1) has a solution defined by the equality

$$\varphi(x) = f(x) + \lim_{\mu\to\infty} \frac{1}{T_\mu} \int_0^{T_\mu} f(\xi) \frac{D(x,\xi;\lambda)}{D(\lambda)} d\xi. \tag{2}$$

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The use of digital tools in teaching the history of mathematics to students of a pedagogical university

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The project "Modern Digital Educational Environment in the Russian Federation" aims higher education to expand the use of information and communication technologies in teaching. The discipline "History of Mathematics" was no exception. Modern digital tools allow you to organize training in different formats. The electronic information educational environment of the university, hosted on the Moodle platform, is a convenient means for implementing any of them. It specifies the sections of the discipline being studied, you can place links to video lectures and recommended literature on each of the topics, materials for independent work, course assignments (tests, essay topics, test papers, etc.), as well as an approximate list of questions submitted for boundary control. The author of this article has created a course of video lectures on the history of mathematics, read by the Doctor of Physical and Mathematical Sciences, Professor of OSPU G.P. Matviyevskaya for future teachers of mathematics. In 2021, a textbook on the history of mathematics was published [1], which is successfully combined with this video course and is actively used in conducting classes for students of the Faculty of Physics and Mathematics of a pedagogical university. In the future, it is planned to prepare a MOOC on the history of mathematics based on the available materials.

In the conditions of clogging the Internet with "raw", unscrupulous publications, which the student is not able to evaluate without sufficient experience and minimal knowledge, obviously, appropriate assistance from the teacher is required. A good source in solving this problem is the website created by associate professor of the SFU V.E. Pyrkov (http://pyrkov-professor.ru /). The richest media library on the history of mathematics is located here. An overview of these materials is presented in the article [2]. Another proven site containing the most valuable books and journal articles on the history of mathematics is https://www.mathedu.ru /, created in the electronic library of RAO by V.M. Busev. You can learn more about the content of the site sections from the article [3]. These sites are necessarily recommended to students at the beginning of studying the history of mathematics course at our university.

For testing students in the history of mathematics, in addition to the Moodle platform, the following can be recommended as working digital tools: MyTest, MENTIMETER, Yandex.Forms, Google.Forms, etc., which enhance the positive result from testing, saving time resources, developing students' self-control and self-education skills, as well as introducing them to life in a high-tech, competitive world.

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About a new approach at classical mechanics

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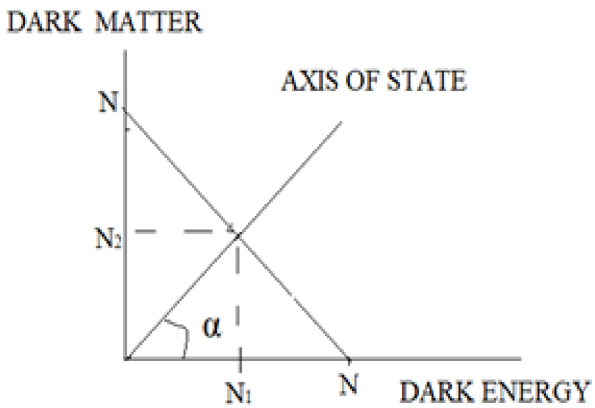
In the dynamics of a material point the principles of classical mechanics are based on *Jacobi's theorem.* If we find the complete integral $S(x, y, z, t, \alpha, \beta, \gamma)$, where α, β, γ are arbitrary non-additive constants for the Jacobi's differential equation

$$\frac{1}{2m}\left[\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2\right]+V\left(x,y,z,t\right)=\frac{\partial S}{\partial t},$$

then the equations $\frac{\partial S}{\partial \alpha} = a$, $\frac{\partial S}{\partial \beta} = b$ and $\frac{\partial S}{\partial \gamma} = c$, where a, b, c are three arbitrary constants, determine one of the possible motions in the force field $V(x, y, z, t)$. Here the momentum components at time t at the point x, y, z are $p_x = mv_x = -\frac{\partial S}{\partial x}$, $p_y = mv_y = -\frac{\partial S}{\partial y}$, $p_z = mv_z = -\frac{\partial S}{\partial z}$. Under the action of the force field V the body has the trajectory of motion. It is clear that the field V is created by matter. For example, the gravity. While studying field, we must also study the matter too. What, how much and how? What are the bodies that act on the experimental body. How many them? How many of them, for example, out of all N bodies, only N_1 of them are those that affect the experimental body, N_2 are not. Today's research by astrophysicists speaks of the existence of dark matter and dark energy. Dark matter binds the body, that is, it attracts it, and this is the gravitation. Dark energy, on the contrary, releases it, pushes it away, that is, does not interact with it. The expansion of the universe proves that today dark energy wins on the stage of the universe. So, let's say we have N bodies and the experimental body is placed in their environment. Will they all affect the body? If so, then one ray of state, the state of the strongest gravitational field that is observed in black holes, is a ray of dark energy. If none of them affects the experimental body, then this is another ray of state, a state of absolute escape from the center - a ray of dark energy. Let's represent them graphically. In today's historical period of the development of the Universe is the its expansion. Therefore, dark energy dominates, but it is not absolute and therefore $\alpha \neq 0°$. There are fluctuations in the Universe, it either shrinks or expands. Due to its expansion, $\alpha < 45°$. But now exact value of α is not fundamental for us in this proposal. So, we consider that arbitrary α, β, γ are N_1, N_2 and α. The number N_1 is "yes" acting on the experimental body, therefore they can be called observable bodies, N_2 is "no" not affecting the body, that is, unobservable bodies and α is the state of the body. The α reflects the probability of a state, for example, the state "no",$p = \frac{N_1}{N}$. Therefore for us $S(x, y, z, t, N_1, N_2,\alpha)$. Let's use the formalism of Jacobi: $\frac{\partial S}{\partial N_1} = a$,$\frac{\partial S}{\partial N_2} = b$ and $\frac{\partial S}{\partial \alpha} = c$. Thus, there are primary constants N_1,N_2 and α, which divide the movement of the body into many classes. For the same N there are different angles α depending on N_1 and N_2. Secondary constants a, b, c choose a needed curve from our one chosen class. We see that number N_1 set the position of the body x, y, z in space. It relates to the force field $V(x, y, z, t)$ that is the potential energy of the body. The number N_2 not tied to gravity, giving freedom and speed to the body is related to the kinetic energy of the body $K = \frac{1}{2}mv^2$. The angle α points the total energy $E, E = V + K = \frac{1}{2}mv^2 + V$. As we know, if the potential energy doesn't depend on time $V \neq V(t)$ then $E = const$. Our α plays role of E. In fact, on the one ray of the state $\alpha = const$ there is no dependence on time $\alpha \neq \alpha(t)$. In this case we can suppose $S(x, y, z, t, N_1, N_2, \alpha) = \alpha t - -S_1(x, y, z, N_1, N_2, \alpha)$, where S_1 is no longer dependent on t.



Substituting into the Jacobi equation we get: $\frac{1}{2m}\left[\left(\frac{\partial S_1}{\partial x}\right)^2 + \left(\frac{\partial S_1}{\partial y}\right)^2 + \left(\frac{\partial S_2}{\partial z}\right)^2\right] +$ $+V(x, y, z, t) = \alpha$, where $\frac{\partial S}{\partial \alpha} = t - \frac{\partial S_1}{\partial \alpha} = c = const = t_0$. Therefore $\frac{\partial S_1}{\partial \alpha} = t - t_0$.

Thus, the secondary constants $a = \frac{\partial S_1}{\partial N_1}$ and $b = \frac{\partial S_2}{\partial N_2}$ determine the trajectory in space. The constant $c = \frac{\partial S_1}{\partial \alpha} = t - t_0$ describes the movement along this trajectory and in it there is only time. In fact, although the particular values of N_1 and N_2 will determine the ray of state for us, the global α will describe to us the changes in the ray itself.

Let's consider the case $V = 0$ no force field, for example, no gravity. Then $\frac{1}{2m}\left[\left(\frac{\partial S_1}{\partial x}\right)^2 + \left(\frac{\partial S_1}{\partial y}\right)^2 + \left(\frac{\partial S_1}{\partial z}\right)^2\right] = \alpha$. We can get $S_1 = \sqrt{2m\alpha}(N_1x + N_2y + (\sqrt{N^2 - N_1^2 - N_2^2}z)$, where $N = N_1 + N_2$. Let's findthe secondary constants $a = \frac{\partial S_1}{N_1} = \sqrt{2m\alpha}(x - \frac{N_1}{\sqrt{1 - N_1^2 - N_2^2}}z)$, $b = \frac{\partial S_1}{N_2} = \sqrt{2m\alpha}(y - \frac{N_2}{\sqrt{1 - N_1^2 - N_2^2}}z)$ and $\alpha = \frac{\partial S_1}{\partial \alpha} = \frac{m}{\sqrt{2\alpha}}(N_1x + N_2y + +\sqrt{N^2 - N_1^2 - N_2^2}\, z =$ $t - t_0$. Then $\alpha = \frac{m^2(N_1x + N_2y + \sqrt{N^2 - N_1^2 - N_2^2}z)^2}{2(t - t_0)^2}$. We see $\alpha = 0$ when 1)$N = 0$, 2) $m = 0$ and 3) $t \to \infty$.

The first case $N = N_1 + N_2 = 0$ means there is no matter in Cosmos. It is before the known us Big Bang. The second case corresponds to one point. It is Big Bang itself. It is before the known us Big Bang. There is a dark energy in the confined space. This space cant be infinity because the multiplication $m \cdot x = 0 \cdot \infty \neq 0$. The second case corresponds to one point. It is Big Bang itself. Here the dark energy shrink into the point, then bang is and the matter and dark matter with it appears. Namely dark matter doesn't allow quarks and hadrons escape into infinite space. It ties them together. Let's say about a time in these two cases. If $t = 0$ then $\alpha = \frac{0}{t_0^2} = 0$. But if $t = \infty$ then $\alpha = \frac{0}{\infty} \neq 0$. That means if there are no a matter and space then there is no a time. The third case corresponds to the photon with a mass equal to 0. The fourth case corresponds an infinite time. Therefore, if there is no force field that binds the body, that is, there is only one dark energy then mass turns into light during finite time $t - t_0$. If mass will not turn into light then there is infinite time t in Cosmos for this act.

The first case $N = N_1 + N_2$=0 means there is no matter in Cosmos. It is before the known us Big Bang. The second case corresponds a photon with a mass equal to 0. The third case corresponds an infinite time. Therefore, if there is no force field that binds the body, that is, there is only one dark energy then mass turns into light during finite time $t - t_0$. If mass will not turn into light then there is infinite time t in Cosmos for this act.

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On basicity of exponential and trigonometric systems of sines and cosines in weighted grand Lebesgue spaces

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The work is devoted to basicity of the system of exponentials and trigonometric systems of sines and cosines in the separable subspace of the weighted space of the grand Lebesgue generated by the shift operator.

Let $\rho : [a, b] \to [0, +\infty)$ be a weight function, $p > 1$, $A_p(a, b)$ be the Mackenhoupt class [1, 2], $L_{p),\rho}(a, b)$ be a weighted grand Lebesgue space [1], i.e. Banach space of measurable on $[a, b]$ functions f with finite norm

$$\|f\|_{p),\rho} = \sup_{0<\varepsilon<p-1} \left(\frac{\varepsilon}{b-a} \int_a^b |f(t)|^{p-\varepsilon} \rho(t) dt \right)^{\frac{1}{p-\varepsilon}}.$$

We denote by $G_{p),\rho}(a, b)$ the closure in $L_{p),\rho}(a, b)$ of the linear manifold of functions f such that $\|T_\delta f - f\|_{p),\rho} \to 0$ for $\delta \to +0$ where T_δ is the shift operator, i.e. $T_\delta f(x) = f(x + \delta)$, $x + \delta \in [a, b]$ and $T_\delta f(x) = 0, \; x + \delta \notin [a, b]$

The following statements are true.

Theorem 1. *Let $\rho \in A_p(-\pi, \pi)$. Then the system of exponents $\left\{e^{\mathrm{int}}\right\}_{n\in Z}$ forms a basis in $G_{p),\rho}(-\pi, \pi)$.*

Theorem 2. *Let $\rho \in A_p(0, \pi)$. Then the system of sines $\{\sin nt\}_{n\geq 1}$ forms a basis in $G_{p),\rho}(0, \pi)$.*

Theorem 3. *Let $\rho \in A_p(0, \pi)$. Then the system of cosines $\{\cos nt\}_{n\geq 0}$ forms a basis in $G_{p),\rho}(0, \pi)$.*

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Frames from weighted exponentials system in grand Lebesgue spaces

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In this paper, we study the frame property of a system $\left\{\mu(t)e^{int}\right\}_{n\in Z}$ in a separable subspace $G_{p)}(0,2\pi)$ of the space of the grand Lebesgue $L_{p)}(0,2\pi)$, with respect to the space of sequences from the coefficients of the basis $\left\{e^{int}\right\}_{n\in Z}$ to $G_{p)}(0,2\pi)$. In particular, the criterion for the frame property of a system $\left\{|t|^{\alpha}e^{int}\right\}_{n\in Z}$ in the space $L_{p)}(0,2\pi)$ ([1]) is generalized to the case $L_{p)}(0,2\pi)$.

Let X be a Banach space and K be a Banach space of sequences of scalars.

Definition. *A* system $\{x_n^*\}_{n\in N}\subset X^*$ is called a frame in X with respect to K (K-frame) if $\forall x\in X$ $\{x_n^*(x)\}_{n\in N}\in K$ and there are numbers $A,B>0$ such that $A\left\|x\right\|_X\leq\left\|\{x_n^*(x)\}_{n\in N}\right\|_K\leq B\left\|x\right\|_X$ *for* $\forall x\in X$.

Let $L_{p)}(0,2\pi)$, $p>1$, be a grand Lebesgue space and $G_{p)}(0,2\pi)$ be a subspace of $L_{p)}(0,2\pi)$ generated by the shift operator T_δ . It is known that ([2]) the system $\left\{e^{int}\right\}_{n\in Z}$ forms a basis in $G_{p)}(0,2\pi)$. Let $K_{p)}$ be the Banach space of sequences of coefficients of expansions of elements $G_{p)}(0,2\pi)$ of in terms of the basis $\left\{e^{int}\right\}_{n\in Z}$ with the norm $\left\|\{a_n\}_{n\in Z}\right\|_{K_{p)}}=\sup\limits_{m,n\in N_0}\left\|\sum\limits_{k=-m}^{n}a_ke^{ikt}\right\|_{G_{p)}(0,2\pi)}$.

Theorem. *The system* $\left\{\mu(t)e^{int}\right\}_{n\in Z}$ *forms a* $K_{p)}$ *frame in* $G_{p)}(0,2\pi)$ *if and only if there are numbers* $c_1,c_2>0$ *such that* $c_1\leq\mu(t)\leq c_2$*, a.e. on* $[0,2\pi]$.

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On uncountable b-Bessel systems in non-separable Hilbert spaces

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In the paper, by means of bilinear mappings we introduce the notion of uncountable b-Bessel systems in non-seperable Hilbert spaces. The criterism of uncountable b- Bessel property is established.

Let X and H be non-seperable Hilbert spaces Y, be a non-seperable Banach spaces. We consider a bilinear mapping $b : X \times Y \to Z$ satisfying the condition

$$\exists M > 0, \|b(x,y)\|_H \leq M \|x\|_X \|y\|_Y, \forall x \in X, \forall y \in Y$$

For $h \in H$ and $y \in Y$ we denote $\omega_b(h,y) \in X : (x, \omega_b(h,y))_X = (b(x,y),h)_H$

Let I be the set of indices equivalent with H and I^a be the set of its at most countable subsets. By $l_2(I^c, X)$ we denote a Hilbert space of the systems $\hat{x} = \{x_\alpha\}_{\alpha \in I}$ of vectors from $X : \{\alpha \in I : x_\alpha \neq 0\} \in I^a$, $\sum\limits_{\alpha \in I} \|x_\alpha\|_X^2 < +\infty$

Definition. We call the system $\{y_a\}_{a \in I} \subset Y$ b-bessel in H, if there exists $\exists B > 0$: $\sum\limits_{\alpha \in I} \|\omega_b(h, y_\alpha)\|_X^2 \leq B \|h\|_H^2, \forall h \in H.$

Theorem. For the system $\{y_a\}_{a \in I} \subset Y$ to be uncountable b -bessel in H, it is necessary and sufficient that the operator determine, by the formula

$$T(\hat{x}) = \sum_{\alpha \in I} b(x_\alpha, y_\alpha)$$

to act boundedly from $l_2(I^c, X)$ to H .

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Simultaneous maximal approximation by Faber partial sums

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In this talk we discuss the simultaneous maximal approximation properties of partial sums of Faber series in the simple connected domains with quasiconformal boundary. As is known the quasiconfomal boundaries may be also non rectifiable and even locally non rectifiable. Although maximal convergence has been adequately researched in the mathematical literature, studies on both maximal convergence and simultaneous approximation are scarcely any. In this study, we will obtain some evaluations about simultaneous maximal approximation speed of partial sums of Faber series in the Bergman space of analytic functions.

On homogeneous Fredholm integral equations in bohr space

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In the article [1] a new integral equations of Fredholm type in Bohr spaces of almost periodic functions were introduced:

$$\varphi(x) = f(x) + \lambda \lim_{T\to\infty} \frac{1}{T} \int_0^T K(x,\xi)\varphi(\xi)d\xi. \tag{1}$$

The analogs of Fredholm functions are defined also. It was proven the analog of first theorem of Fredholm.

Here we consider the homogeneous integral equations, i. e. in (1) we put $f(x) \equiv 0$. Denote by $D(\lambda), D(x,y;\lambda)$ the analogs of Fredholm function introduced in [1]. The following is an analog of the second theorem of Fredholm.

Theorem 1. Let λ_0 be a real root of the function $D(\lambda)$ and $g_l(x,y)$ be a first nonzero coefficient of Taylor expansion for the function $D(x,y;\lambda)$. Then $g_l(x,y)$ for some fixed real values of y will be a nonzero solution for homogeneous equation.

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Calculation algorithm of probability of A Lukaseiwicz's intuitionistic propositional factor formula according to the truth table.

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If $\mathcal{A}$ Lukaseiwicz's intuitionistic propositional factor formula as statement letters $A_1, \ldots, A_n$ and the factors $A_1, \ldots, A_n$ are mutually independent. Clearly $\mathcal{A}$ can be represented by a truth table of 3^n rows, where each row represents some assignment of truth values to the variables $A_1, \ldots, A_n$, followed by the corresponding value of $\mathcal{A}\left(A_1, \ldots, A_n\right)$.

Let's calculate $z_i = k_1^i \cdot \ldots \cdot k_n^i$ for each row ($1 \leq i \leq 3^n$). Here, if variable of $A_j\, (j = 1, \ldots, n)$ gets t(true) at the row i, then $k_j^i = p_j$,if variable of $A_j \;\; (j = 1, \ldots, n)$ gets f (false), then $k_j^i = q_j$, if variable of $A_j \;\; (j = 1, \ldots, n)$ gets n (neutral) , then $k_j^i = n_j$. Let's define $Pt/\mathcal{A})$ as the sum of rows z_i where $\mathcal{A}$ factor formula gets t (true) values at truth table. If there is no such row, then we accept $P(t/\mathcal{A}) = 0$. Let's define $P(f/\mathcal{A})$ as the sum of rows z_i $\mathcal{A}$ factor formula gets f (false) values at truth table. If there is no such row, then $P(f/\mathcal{A}) = 0$. Let's define $P(n/\mathcal{A})$ as the sum of rows z_i where $\mathcal{A}$ factor formula gets n (neutral) values at truth table. If there is no such row, then $P(n/\mathcal{A}) = 0$.

$$(p_1 + q_1 + n_1) = 1, \ldots, (p_{3^n} + q_{3^n} + n_{3^n}) = 1$$

and

$$P(t/\mathcal{A}) + P(f/\mathcal{A}) + P(n/\mathcal{A}) = (p_1 + q_1 + n_1) \ldots (p_{3^n} + q_{3^n} + n_{3^n})$$

It is clear $P(t/\mathcal{A}) + P(f/\mathcal{A}) + P(n/\mathcal{A}) = 1$ If clear $\mathcal{A}$ and $\mathcal{B}$ logically equivalent then:
$P(t/\mathcal{A}) = P(t/\mathcal{B}); P(f/\mathcal{A}) = P(f/\mathcal{B}); P(n/\mathcal{A}) = P(n/\mathcal{B})$

The obtained results can be generalized for k-valued logics ($k > 3$). The results can be use in machine learning, crowd-sourcing and -sensing, information flow, information theory and systems neuroscience.

On the time of corrosion failure of a plate with a deep double-sided external undercut under pure bending

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Let a plate with a deep two-sided external symmetrical undercut be bent by a moment M in an aggressive environment. In this case, the plate is subjected to stress corrosion failure. We assume that its initial thickness d_o decreases in time t. The maximum stress in the plate occurs at the bottom of the undercut and has the following expression [1]:

$$\sigma_{max} = \frac{M}{2\alpha^2 d_o} \frac{4\frac{\alpha}{\rho}\sqrt{\frac{\alpha}{\rho}}}{\left[\sqrt{\frac{\alpha}{\rho}} + \left(\frac{\alpha}{\rho} - 1\right) arctg\sqrt{\frac{\alpha}{\rho}}\right]} \tag{1}$$

Here $\frac{\alpha}{\rho}$ is the ratio of half the width of the narrowest part of the plate to the radius of curvature. In the process of corrosion, the dimensions of the plate, including the thickness d, will undergo changes: $d = d(t)$. The problem is to determine the dependence of the area $S(t) = ad(t)$ of the narrowest section of the plate on time t. For the rate of the corrosion process, we use the well-known Glickman formula [2]:

$$\frac{ds(t)}{dt} = -k - n\sigma_{max}; \qquad s(t)|_{t=o} = s_o, \tag{2}$$

where k, n are experimentally determined universal constants of the "metal-aggressive environment" system. Taking into account (1) in the first relation (2) and using the second relation (2), the dependence between the time t and the value s was obtained. From this dependence, the time of complete depletion ($s = 0$) of the plate is determined:

$$t_{o\sigma} = \frac{s_o}{k} + \frac{2n\sqrt{\frac{\alpha}{\rho}}\left(1 + \frac{\alpha}{\rho}\right)M}{k^2\left[\sqrt{\frac{\alpha}{\rho}} + \left(\frac{\alpha}{\rho} - 1\right) arctg\sqrt{\frac{\alpha}{\rho}}\right]} \times$$

$$\times \ln\left|\frac{2n\sqrt{\frac{\alpha}{\rho}}\left(1 + \frac{\alpha}{\rho}\right)M}{k^2 s_o\left[\sqrt{\frac{\alpha}{\rho}} + \left(\frac{\alpha}{\rho} - 1\right) arctg\sqrt{\frac{\alpha}{\rho}}\right] + 2nM\sqrt{\frac{\alpha}{\rho}}\left(1 + \frac{\alpha}{\rho}\right)}\right|.$$

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Asymptotics of eigenvalues and eigenfunctions of a discontinuous boundary value problem with a spectral parameter in the transmission condition

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Consider the following boundary value problem:

$$-y^{''}=\lambda y,\quad x\epsilon[0,\frac{q}{p})\cup(\frac{q}{p},1], \tag{1}$$

$$\left.\begin{array}{c} y\left(0\right)=y\left(1\right)=0 \\ y\left(\frac{q}{p}-0\right)=y\left(\frac{q}{p}+0\right) \\ y^{'}\left(\frac{q}{p}-0\right)-y^{'}\left(\frac{q}{p}+0\right)=\lambda my\left(\frac{q}{p}\right) \end{array}\right\}, \tag{2}$$

here $q\in N, p\in\mathbb{Z}_{+},\ \ q<p,\ 0\neq m\in\mathbb{C}$.

It is proved the following theorem:

Theorem. *Eigenvalues of the problem (1)-(2) are asymptotically simple and consist of union of $p-q+1$ number of sequences $\{\lambda_{1,n}\}_{n=0}^{\infty}, \{\lambda_{2,\ \ n}\}_{n=0}^{\infty}$ and $\{\lambda_{l.\ n+1}\}_{n=0}^{\infty}, l=3,4,\ldots,p-q+1$; the following equalities hold:*

$$\sqrt{\lambda_{1,n}}=p\pi n, \sqrt{\lambda_{2,n}}=\frac{p\pi}{q}n+\frac{1}{\pi mn}+o\left(\frac{1}{n^2}\right),$$

$$\sqrt{\lambda_{l,\ \ n+1}}=p\pi n+\frac{p}{p-q}\pi\left(l-2\right)+\frac{1}{m\left(p-q\right)\pi n}+o\left(\frac{1}{n^2}\right),\ 3\leq l\leq p-q+1,$$

as $n\to\infty$.

For the eigenfunctions $y_{1,n}\left(x\right),\ \ y_{2,n}\left(x\right)$ and $y_{l,n+1}\left(x\right)$, that correspond to eigenvalues $\lambda_{1,n}=\rho_{1,n}^2$, $\lambda_{2,n}=\rho_{2,n}^2$ and $\lambda_{l,n+1}=\rho_{l,n+1}^2$, respectively, the following equalities hold:

$$y_{1,n}\left(x\right)=\mathrm{sin}p\pi nx,\quad x\in\left[0,1\right],\ n=1,2,\ldots$$

$$y_{2,n}\left(x\right)=\left\{\begin{array}{c} \gamma_{2,n}^{'}sin\frac{p\pi n}{q}x+O\left(\frac{1}{n}\right),\quad x\in[0,\frac{q}{p}) \\ \\ \gamma_{2,n}^{''}sin\frac{p\pi n}{q}(x-1)+O\left(\frac{1}{n}\right),\quad x\in(\frac{q}{p},\ 1] \end{array}\right.$$

$$y_{l,n+1}\left(x\right)=\left\{\begin{array}{c} \gamma_{l,n+1}^{'}\sin(p\pi n+\frac{p}{p-q}\pi\left(l-2\right))x+O\left(\frac{1}{n}\right),\quad x\in[0,\frac{q}{p}) \\ \\ \gamma_{l,n+1}^{''}\sin(p\pi n+\frac{p}{p-q}\pi\left(l-2\right))(x-1)+O\left(\frac{1}{n}\right),\quad x\in(\frac{q}{p},\ 1] \end{array}\right.,$$

here

$$\gamma'_{2,n} = -\cos\left(\frac{p-q}{q}\pi n\right) + O\left(\frac{1}{n}\right), \gamma''_{2,n} = O\left(\frac{1}{n}\right),$$

$$\gamma'_{l,n+1} = (-1)^{pn-1+l} + O\left(\frac{1}{n}\right)$$

$$\gamma''_{l,n+1} = -\cos\left(\frac{q}{p-q}\pi\left(l-2\right)\right) +$$

$$+m\left(p\pi n + \frac{p}{p-q}\pi\left(l-2\right)\right)\sin\left(\frac{q}{p-q}\pi\left(l-2\right)\right) + O\left(\frac{1}{n}\right).$$

The above problem in special cases were studied in [1,2].

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Ill-posed problems for systems of first-order elliptic type equations

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In this article, we are talking about solving the ill-posed problem for matrix factorizations of the Helmholtz equation [2]. This study concerns the ill-posed problems, i.e., it is unstable. It is known that the Cauchy problem for elliptic equations is unstable relatively small change in the data, i.e., incorrect [1]. In unstable problems, the image of the operator is not closed, therefore, the solvability condition can not be written in terms of continuous linear functionals. In conditionally correct problems, problems that are correct in the sense of A.N. Tikhonov, we are no longer just talking about a solution, but about a solution that belongs to a certain class. Narrowing the class of admissible solutions allows in some cases to pass to the well-posed problem. The first results in this direction appeared only in the mid-1980s in the works of L.A. Aizenberg, A.M. Kytmanov, N.N. Tarkhanov [1].

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Stabilization of solutions to chevron pattern equations

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The lak will be devoted to the problem of global stabilization of solutions to the initial boundary value problem for the following system of equations modeling the evolution of chevron patterns:

$$\tau\partial_t A = A + \Delta A - \phi^2 A - |A|^2 A - 2ic_1\phi\partial_y A + i\beta A\partial_y\phi, \tag{1}$$

$$\partial_t\phi = D_1\partial^2\phi + D_2\partial^2\phi - h\varphi + +\varphi|A|^2 - c_2 Im[A^*\partial_y A \tag{2}$$

where τ, D_1, D_2, c_1,c_2, h are non-negative constants and $\beta \in R$. The complex valued function A (where A^* denotes its complex conjugate) succinctly represents the phase (clockwise/ counter-clockwise), direction and amplitude (wave vector) of the periodical patterns, whereas the orientation of the liquid crystals is represented via the real valued function ϕ, the angle of the director vector (projected in the $x - y$ plane) with the x-axis. The parameter τ is a function of the various physical time-scales of the problem, and D_1 and D_2 are the coefficients of the anisotropic diffusion of the director field for the liquid crystal particles. The rest of the parameters reflect various interactions:

-the dampening parameter h measures the tendency of the director field to align with the magnetic field $\vec{H}$, corresponding to $\phi = 0$,

-the parameter β measures the interaction between the gradient of the director field and the phase of the rolls, and

-the parameters c_1 and c_2 control the torque that the director field and the wave vector of the rolls exert on each other; when $c_1 = c_2 = 1$ the interaction is isotropic, but many experimentally interesting phenomena occur in the anisotropic regime.

On free and independent works of students in the teaching of mathematical

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The formation of educational outcomes takes into account the four main goals of higher education: successful entry into the labor market, gaining an active life position, personal development, supporting a wide range of advanced knowledge. Like other professionals, teachers need to meet these requirements.

Mathematical analysis is one of the main subjects that will help future teachers to achieve these goals. The increase in the flow of information, the inclusion of new subjects in the curriculum leads to reduction in teaching hours in this subject. The performance of students' free and independent works is of special importance for the study of the subject at the required level.

Resolution No. 348 of the Cabinet of Ministers of the Republic of Azerbaijan dated December 24, 2013 states that the hours allocated for free and independent works are determined not less than 1 (one) hour for 1 (one) academic hour of all classes, 40% of which is carried out under the guidance of a teacher [1]. The student's independent work is carried out under the guidance of the teacher outside the classroom and independently. The teacher-led section consists of reviewing current advice, essays and homework, as well as giving assignments and recommendations on mastering the subject.

The report discusses the experience of conducting free and independent works on the subject of "Mathematical Analysis".

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On some properties of the Riesz potential in the grand Lebesgue and grand Sobolev spaces

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The work considers the Riesz-type potential in non-standard grand-Lebesgue and grand-Sobolev spaces. The classical facts about Lebesgue and Sobolev spaces carry over to this case.

We adopt the following standard notation. R^n is an n-dimensional Euclidean space; Everywhere in what follows, p' means the number conjugate of p, i.e. $\frac{1}{p}+\frac{1}{p'}=1$.

Let us define the grand Lebesgue space. Let $\Omega\subset R^n$ be a bounded domain with Lebesgue measure $|\Omega|$. The Grand Lebesgue space $L_{p)}(\Omega)$, $1<p<+\infty$, is the Banach space of measurable functions (according to Lebesgue) on Ω with norm

$$\|f\|_{p)}=\sup_{0<\varepsilon<p-1}\left(\frac{\varepsilon}{|\Omega|}\int_\Omega |f|^{p-\varepsilon}\,dx\right)^{\frac{1}{p-\varepsilon}}.$$

Denote the grand-Sobolev space generated by this norm by $W^m_{p)}(\Omega)$. The norm in $W^m_{p)}(\Omega)$ is given by

$$\|f\|_{W^m_{p)}}=\sum_{|\alpha|=0}^{m}\|\partial^\alpha f\|_{p)}.$$

These spaces are not separable. We have the following continuous embeddings

$$L_q(\Omega)\subset L_{q)}(\Omega)\subset L_{q-\varepsilon}(\Omega)\,,\ \ \forall\varepsilon\in(0,q-1)\,.$$

Regarding these and other facts, more details can be found in the works [1, 2, 1, 4, 5].

Let $\Omega\subset R^n$ be a bounded domain, $A(\cdot;\cdot)\in C\left(\bar\Omega\times\bar\Omega\right)$ be a continuous function. Consider the following Riesz-type integral

$$(K\rho)(x)=u(x)=\int_\Omega\frac{A(x;y)}{r^\lambda}\rho(y)\,dy, \tag{1}$$

where $r=|x-y|$ is a distance between points $x;y\in\bar\Omega$, $\lambda\in[0,n)$ is some number.

The following theorem is true.

Theorem 1. *Let $\Omega\subset R^n$ be a bounded domain, $\rho\in L_{q)}(\Omega)$, $A\in C\left(\overline{\Omega}\times\overline{\Omega}\right)$ and $\lambda q'<n$. Then the operator (1) acts completely continuously from $L_{q)}(\Omega)$ to $C\left(\overline{\Omega}\right)$.*

The following theorem is true.

Theorem 2. *Let $\lambda q^{'}\geq n$ and an integer s satisfy $n-(n-\lambda)\,q<s\leq n$. Then the integral (1) defines a function that, on any section Ω_s of the set Ω by an s-dimensional plane, is defined almost everywhere in the sense of the Lebesgue measure in R^s. The operator K, defined by the formula (1), is bounded as an operator from $L_{q)}(\Omega)$ to $L_r(\Omega_s)$ (and hence to $L_{r)}(\Omega_s)$), for $\forall r$: $1<r<r_0=\frac{sq}{n-(n-\lambda)q}$.*

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Formula for calculating the normal derivative of the double layer potential

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It is known that in two-dimensional space the boundary-value problems for the Helmholtz equation are reduced to a singular integral equation depending on the operator

$$(Tf)(x) = 2\frac{\partial}{\partial v(x)}\left(\int\limits_L \frac{\partial \Phi(x,y)}{\partial v(y)} f(y)dL_y\right),\ \ x\in L,$$

which is generated by the normal derivative of the double layer potential, where $L \subset R^2$ is a simple and closed Lyapunov curve, f is a continuously differentiable function on L, $v(x)$ is a unit outward normal at the point $x \in L$, $\Phi(x,y) = \frac{i}{4}H_0^{(1)}(k|x-y|)$ is a fundamental solution of the Helmholtz equation $\triangle u + k^2 u = 0$, k is the wave number, and $Imk \geq 0$, and $H_0^{(1)}$ is Hankel's function of the first kind of zero order. Obviously, in order to study the solution of the obtained integral equations, the development of a practical formula for calculating the normal derivative of the double layer potential is of paramount importance, to which the present work is dedicated.

The following theorem holds.

Theorem 1. Let

$$\int\limits_0^{diamL} \frac{\omega(gradf,t)}{t}dt < +\infty.$$

Then the double layer potential has a normal derivative on L, and

$$(Tf)(x) = 2\int\limits_L \frac{\partial V(x,y)}{\partial v(x)} f(y)dL_y -$$

$$-\frac{2}{\pi}\int\limits_L \frac{(x-y,v(y))(x-y,v(x))}{|x-y|^4}(f(y)-f(x))dL_y +$$

$$+\frac{1}{\pi}\int\limits_L \frac{(v(y),v(x))}{|x-y|^2}(f(y)-f(x))dL_y,\ \ x\in L, \quad (1)$$

where $\omega(\phi,\delta)$ is the modulus of continuity of the function $\phi(x) \in C(L)$,

$$V(x,y) = \left(\frac{i}{4} - \frac{C}{2\pi} - \frac{1}{2\pi}\ln\frac{k|x-y|}{2}\right)(y-x,v(y))\times$$

$$\times \sum_{m=1}^{\infty} \frac{(-1)^m k^{2m} |x-y|^{2m-2}}{2^{2m-1}(m-1)!m!} -$$

$$-(y-x, v(y)) \sum_{m=1}^{\infty} \left(\sum_{m=1}^{\infty} \frac{1}{l} \right) \frac{(-1)^{m+1} k^{2m} |x-y|^{2m-2}}{2^{2m+1}(m-1)!m!} -$$

$$-\frac{1}{2\pi}(y-x, v(y)) \sum_{m=1}^{\infty} \frac{(-1)^m k^{2m} |x-y|^{2m-2}}{2^{2m}(m!)^2},$$

C is an Euler's constant and the last integral in equality (1) exists in the sense of the principal Cauchy value.

Convergence of the spectral expansion of a function from the class $W^1_{(p,m)}(G), 1 < p < 2$, in the vector eigenfunctions of a differential operator of the third order

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In the interval $G = (0, 1)$, we consider a differential operator

$$L\Psi = \Psi^{(3)} + U_1(x)\Psi^{(2)} + U_2(x)\Psi^{(1)} + U_3(x)\Psi$$

with matrix coefficients $U_l(x) = (u_{lij}(x))_{i,j=1}^m, l = \overline{1,3}$, where $u_{lij}(x) \in L_1(G)$ are complex-valued functions.

Assume that $\{\Psi_k(x)\}_{k=1}^\infty$ is the complete system of vector eigenfunction of the operator L orthonormal in $L_2^m(G)$. By $\{\lambda_k\}_{k=1}^\infty$ we denote the corresponding system of eigenvalues. Moreover, we assume that $Re\ \lambda_k = 0$. Parallel with the spectral parameter λ_k, we consider a parameter μ_k :

$$\mu_k = \begin{cases} (-i\lambda_k)^{1/3}, & for\ \ Im\ \lambda_k \geq 0, \\ (i\lambda_k)^{1/3}, & for\ \ Im\ \lambda_k < 0 . \end{cases}$$

We now introduce a partial sum of the orthogonal expansion of the vector function $f(x) \in W^1_{p,m}(G)$ in the system $\{\Psi_k(x)\}_{k=1}^\infty$:

$$\sigma_\nu(x, f) = \sum_{\mu_k \leq \nu} f_k \Psi_k(x), \quad \nu > 0,$$

where

$$f_k = (f, \Psi_k) = \int_0^1 < f(x), \Psi_k(x) > dx = \int_0^1 \sum_{l=1}^m f_l(x)\overline{\psi_{kl}}(x)\, dx,$$

$$\Psi_k(x) = (\Psi_{k1}(x), \Psi_{k2}(x), \dots, \Psi_{km}(x))^T,$$

and the difference $R_\nu(x, f) = f(x) - \sigma_\nu(x, f)$.

Theorem. Suppose that $U_1(x) \in L_2(G), U_r(x) \in L_1(G), r = \overline{2,3}$; $f(x) \in W^1_{p,m}(G), 1 < p < 2$ condition

$$\left|\left\langle f(x), \Psi_k^{(2)}(x)\right\rangle|_0^1\right| \leq C_1(f)\,\mu_k^\alpha \|\Psi_k\|_{\infty,m} ,$$

where $0 \leq \alpha < 2$, $\mu_k \geq 1$, $C_1(f)$ is a constant depending on $f(x)$, is satisfied and the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ is uniformly bounded.

Then the spectral expansion of the vector function c in the system $\{\Psi_k(x)\}_{k=1}^{\infty}$ absolutely and uniformly converges on $\overline{G} = [0,\ 1]$ and the following estimate is true:

$$\|R_\nu(\cdot, f)\|_{C[0,1]} \leq C\left\{C_1(f)\,\nu^{\alpha-2} + \nu^{-\frac{1}{2}}\|U_1^* f\|_{2,m} + \nu^{-\frac{1}{q}}\left\|f'\right\|_{p,m} + \right.$$

$$\left. + \nu^{-1}\|f\|_{\infty,m}\sum_{r=2}^{3}\nu^{2-r}\|U_r\|_1\right\},\ \nu \geq 2,$$

where $p^{-1} + q^{-1} = 1$ and the constant C is independent of $f(x)$.

Corollary.

1.If the vector function f in Theorem satisfies the condition $f(0) = f(1) = 0$ then condition (1) is definitely satisfied and the following estimate is true:

$$\|R_\nu(\cdot, f)\|_{C[0,1]} \leq C\left(\nu^{-\frac{1}{2}}\|U_1^* f\|_{2,m} + \nu^{-\frac{1}{2}}\left\|f'\right\|_{p,m}\right),\ \nu \geq 2,$$

where the constant C is independent of f.

2.If in Theorem $C_1(f) = 0$ or $0 \leq \alpha < 2 - q^{-1}$, then

$$\|R_\nu(\cdot, f)\|_{C[0,1]} = o\left(\nu^{-\frac{1}{q}}\right),\ \nu \longrightarrow +\infty,$$

where the symboll o depends on f.

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Optimization of Higher-order differential inclusions

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The paper deals with the Mayer problem of the higher-order retarded evolution differential inclusions (DFIs) with endpoint constraints:

$$\begin{aligned} &\text{minimize } f\big(x(0), x(T)\big),\ \tfrac{d^s x(t)}{dt^s} \in F\big(x(t), x(t-\tau), t\big),\ \text{a.e. } t \in [0, T],\\ (HRD) \quad &x(t) = \varphi(t),\ t \in [-\tau, 0),\ \big(x^{(j)}(0), x^{(j)}(T)\big) \in Q_j,\ j = 0, 1, ...s-1, \end{aligned}$$

where $F(\cdot, t) : \mathbb{R}^{2n} \rightrightarrows \mathbb{R}^n$ is a set-valued mapping [1]-[6].
Theorem 1. *Suppose that there exists a triple of functions* $\{x^*(\cdot), \psi^*(\cdot), \tilde{x}(\cdot)\}$ *satisfying a.e. the Euler-Lagrange type inclusions and transversality conditions. Then the trajectory* $\tilde{x}(\cdot)$ *is optimal in the convex problem (HRD).*

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On distribution of zeros of first kind modified Bessel function and its derivative

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In the works [1,2] it is shown that modified Bessel function of order ν, has no zeros in the right half-plane. Analysis of ν, zeros of the linear combination of the function $I_\nu(z)$ itself and its derivative $\frac{d}{dz}I_\nu(z)$ is of interest. However, the technique used in [1,2] does not allow to study the zeros of the linear combination of the function $I_\nu(z)$ and its derivative $\frac{d}{dz}I_\nu(z)$.

Let us consider the modified Bessel equation

$$z^2u'' + zu' - \left(z^2 + \nu^2\right)u = 0, \tag{1}$$

where ν is a complex parameter. It is well known that this equation has the solution $I_\nu(z)$, representable in the form

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!\Gamma(\nu+k+1)}, \tag{2}$$

where $\Gamma(\cdot) -$ is a gamma function. The function $I_\nu(z)$ is called a modified Bessel function of first kind.

In the present paper we study distribution of the zeros of the function $aI_\nu(z) + b\frac{dI_\nu(z)}{dz}$, considered as a function of order ν, where a and $b$one real, and $z > 0$. The involvement of some spectral problem in this issue is a facture of this paper. The following theorem is the main result of this paper.

Theorem. *For each fixed $z > 0$ and for aby real a and b, where $a^2 + b^2 > 0$, $ab \geq 0$, the function $aI_\nu(z) + b\frac{dI_\nu(z)}{dz}$ has no zeros in the right half-plane $Re\nu \geq 0$.*

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On approximation in weighted generalized grand Smirnov classes

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Let $G \subset \mathbb{C}$ be a finite Jordan domain with a regular boundary. In this work, we investigate the approximation problems in weighted grand Smirnov classes. When the weight function satisfies the Muckenhoupt condition we prove a direct theorem for polynomial approximation of functions in certain subclasses $\widetilde{E}^{p),\theta}(G)$, $1 < p < \infty$, $\theta > 0$ of weighted grand Smirnov classes [2]. The direct theorem is proved in terms of the modulus smoothness [1]. Also, from the main theorem some results are obtained. In the proof of the main theorem, a Jackson-Dzjadyk polynomial is used.

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Behavior of solution of nonlinear third order equation with nonlinear boundary conditions

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On a smooth domain $\Omega \subset \mathbb{R}^n$ with boundary $\partial\Omega$, we consider the following problem

$$u_{tt} - \alpha\Delta u - \beta\Delta u_t + \gamma u_t + f(u) = 0, \quad (x,t) \in \Omega \times [0,T] \tag{1}$$

$$u(x,0) = u_0(x),\ u_t(x,0) = u_1(x),\ x \in \Omega \tag{2}$$

$$\frac{\partial u}{\partial n} + \frac{\partial u_t}{\partial n} = g(u), \quad (x,t) \in \partial\Omega \times [0,T] \tag{3}$$

where $f(u)$ and $g(u)$ are some nonlinear functions, α, β, γ are positive numbers, $\Delta u = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ Laplace's operator, $\frac{\partial}{\partial n}$ the external derivative on normal at $\partial\Omega$.

A problem of behavior of solutions for equations (1) with different boundary conditions were devoted a lot of papers (see [1],[2] and their references).

In this work, we study stabilizations of solution for a problem (1)-(3), when boundary function has some smoothing properties.

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About one integral representation of functions in Sobolev's anisotropic space

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The first integral representation of functions of several variables defined in regions (star-like, with respect to a point of some ball) $G \subset R^n$ belongs to S. L. Sobolev. Various integral representations for functions of several variables from Sobolev spaces with dominant mixed derivatives have been developed by many mathematicians and have received new interesting and important applications [1–6]. In this paper, for the first time, one integral representation is found for functions of three variables in one anisotropic Sobolev space with a dominant mixed derivative. The result obtained is used in the study of three-dimensional non-classical boundary value problems for equations with a dominant mixed derivative $D_1 D_2 D_3\ u$ with L_p-coefficients in the space of S. L. Sobolev $W_p^{(1,1,1)}(G)$ .Let $G_\tau = (x_\tau^0, h_\tau), (\tau = 1,2,3)$, $G = G_1 \times G_2 \times G_3$, $W_p^{(1,1,1)}(G) = \left\{ u(x) \in L_p(G) / D_1^{i_1}\, D_2^{i_2}\, D_3^{i_3} u \in L_p(G);\ \ i_\zeta = 0,1; \zeta = 1,2,3 \right\}$, $1 \le p \le \infty$, where $x = (x_1, x_2, x_3) \in G, D_\tau = \partial / \partial x_\tau,\ (\tau = 1,2,3)-$ operator of generalized differentiation in the sense of S. L. Sobolev. We define the norm in it by the equality

$$\|u\|_{W_p^{(1,1,1)}(G)} = \sum_{i_1, i_2, i_3 = 0}^{1} \left\| D_1^{i_1}\, D_2^{i_2}\, D_3^{i_3} u \right\|_{L_p(G)}.$$

Theorem. *If $u(x) \in W_p^{(1,1,1)}(G)$, then the function $u(x)$ can be represented as*

$$u(x) = u\left(\frac{x_1^0 + h_1}{2}, \frac{x_2^0 + h_2}{2}, \frac{x_3^0 + h_3}{2}\right) + \int_{\frac{x_1^0 + h_1}{2}}^{x_1} D_1 u(\tau_1, \frac{x_2^0 + h_2}{2}, \frac{x_3^0 + h_3}{2}) d\tau_1 +$$

$$+ \int_{\frac{x_2^0 + h_2}{2}}^{x_2} D_2 u\left(\frac{x_1^0 + h_1}{2}, \tau_2, \frac{x_3^0 + h_3}{2}\right) d\tau_2 + \int_{\frac{x_3^0 + h_3}{2}}^{x_3} D_3 u\left(\frac{x_1^0 + h_1}{2}, \frac{x_2^0 + h_2}{2}, \tau_3\right) d\tau_3 +$$

$$+ \int_{\frac{x_1^0 + h_1}{2}}^{x_1} \int_{\frac{x_2^0 + h_2}{2}}^{x_2} D_1 D_2 u\left(\tau_1, \tau_2, \frac{x_3^0 + h_3}{2}\right) d\tau_1 d\tau_2 +$$

$$+ \int_{\frac{x_2^0 + h_2}{2}}^{x_2} \int_{\frac{x_3^0 + h_3}{2}}^{x_3} D_2 D_3 u\left(\frac{x_1^0 + h_1}{2}, \tau_2, \tau_3\right) d\tau_2 d\tau_3 +$$

$$+ \int_{\frac{x_1^0 + h_1}{2}}^{x_1} \int_{\frac{x_3^0 + h_3}{2}}^{x_3} D_1 D_3 u\left(\tau_1, \frac{x_2^0 + h_2}{2}, \tau_3\right) d\tau_1 d\tau_3 +$$

$$+ \int_{\frac{x_1^0 + h_1}{2}}^{x_1} \int_{\frac{x_2^0 + h_2}{2}}^{x_2} \int_{\frac{x_3^0 + h_3}{2}}^{x_3} D_1 D_2 D_3 u(\tau_1, \tau_2, \tau_3) d\tau_1 d\tau_2 d\tau_3.$$

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Long-term torsional strength of a damaged hollow shaft

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Hollow shafts and pipes are one of the most common structural elements, machines and mechanisms and one of the basic types of loading is a torque transmitted by them.

Analysis of life cycle of twisted hollow shafts represents an important stage in the overall process of calculation and design of constructions, machines and mechanisms. In modern stage of development of engineering, this calculation involves taking account new factors. One of the most important ones is damageability of the material, i.e. initiation and accumulation of various nature defects. In the presence of this phenomenon, destruction is a process in time consisting of the latent incubation period of destruction and apparent destruction stage accompanied by formation and extension of the zone of the destructed part of the material. In this aspect, we consider a problem of a long-term destruction of a hollow isotropic shaft twisted by a constant moment based on the hereditary theory of damageability.

A problem of initiation and development of scattered destruction zone in a twisted hollow shaft with regard to phenomenon of damageability of its material, was solved. An equation of motion of destruction front was obtained and solved. Regularities of torsional scattered destruction of a hollow shaft were determined. The curve of destruction front motion was structured.

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Corrosive failure of a hollow cone under the action of torque

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Corrosive “ mechanical failure is one of the dangerous varieties of failure of metallic structural elements. Among of them a conjecture suggested by Yu.N.Rabotnov on which basis he derived a formula for a time to corrosive failure under constant load (variable stress) and constant deformation.

$$t_* = A\{\frac{\sigma_{eo}}{\sigma_s}(\lambda - atctg\sqrt{\frac{\sigma_{eo}}{\sigma_s} - 1}) - \sqrt{\frac{\sigma_{eo}}{\sigma_s} - 1}\}.$$

Here $\lambda = \sqrt{\frac{\sigma_s}{\sigma_e}(1 - \frac{\sigma_s}{\sigma_e})} + arctg\sqrt{\frac{\sigma_b}{\sigma_s} - 1} = const.$

The author derived formula for the time to corrosive failure based on alternative approach of conception of accumulated damages. It was established that this formula is true regardless failure mechanism of metal in corrosive medium. Using formula, we determine the time to corrosive failure of a conic shaft under the action of torque. To determine the place where the corrosive failure will initiate, we find the extremum of the function t_*. Corresponding calculations show that at first corrosive failures of the cone under consideration initiate from its external surface.

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Compactness of weighted endomorphism of functional algebras of A -valued functions on local compacted sets

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Let X be a compact Hausdorff space and $A = A(X)$ is a uniform algebra defined on the compact$X$. By $C(X,\ A)$ we denote the topological space of all continuous A-valued functions on X with compact-open topology. It is assumed that the topological space X is completely regular with respect to topological algebras (in Tikhonov's sence), i.e. for any closed subset $K \subset X$ and for any point $x \in X \backslash K$ there exist continuous functions$f, g \in C(X)$, such that

$$f\,|_K \equiv 0,\ \ f(x) = 1,\ \ g\,|_K \equiv 1,\ \ g(x) = 0.$$

The notions of peak point and a peak set are defined by following way:

- $x \in X$ is a peak point of A: if there exists the sequence $\{f_n\} \subset A(X)$, such that for arbitrary neighborhood $O(x)$ of $x \Rightarrow f_n\left|_{X \backslash O(x)} \to 0\right.$ uniformly;

- $E \subset X$ is a peak set of A: if there exists the sequence $\{f_n\} \subset A(X)$, such that for arbitrary neighborhood $O(E)$of $E \Rightarrow f_n\left|_{X \backslash O(E)} \to 0\right.$ uniformly.

We denote the set of all peak points by $S_0(A)$ and the set of all peak sets by$S(A)$. The weighted composition operator $T : A \to C(X, A)$ is defined as

$$(Tf)(x) = (u * (f \circ u))(x),\ \ f \in C(X, A),$$

where $u \in C(X, A)$ is a fixed A-valued function and $\varphi : X \to X$ is a self-mapping of X, which is continuous on the support set $X_u$of u, $X_u = \{x : u(x) \neq 0\}$.

In our previous work the compactness of weighted endomorphism is investigated. In the present work we consider the case when X is a local connected compact. In this case the following theorem is true.

Theorem. *Let X be a local compact set. $C(X, A)$ is a topological space of all continuous A-valued functions with compact-open topology. $A = A(X)$ is a subspace of $C(X, A)$. $T : A \to C(X, A)$: $T = (u * f) \circ \varphi$ is a weighted composition operator. Then for compactness of the operator T it is necessary and sufficient: for each compact connected component $K \subset X_u$ either $\varphi(K)$ is finite set, or $\varphi(K) \subseteq X \backslash S_0(A)$.*

The regularized trace formula for the "Weighted" Sturm-Liouville equation point δ-interaction

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In this study, we consider a boundary-value problem for the differential equation

$$-y^{''} + q(x) = \lambda\rho(x)y \qquad x \in (0, \pi), \tag{1}$$

with the boundary conditions

$$U(y) := y^{'}(0) = 0, \quad V(y) := y(\pi) = 0, \tag{2}$$

when $\rho(x) = r(x) + \alpha\delta(x - a)$, where $\delta(x)$ is the Dirac function (see[1]), $q(x)$ is real-valued function in $W_2^1(0, \pi)$ and $\alpha > 0; r(x)$ is piecewise constant function:

$$r(x) = \begin{cases} r^2, & 0 \le x < a \\ 1, & a \le x \le \pi, \end{cases} \quad 0 < r \ne 1. \tag{3}$$

Here, λ is a spectral parameter.

After construction of the Hilbert space related to (1), we obtain the formula of the first order regularized trace for the boundary-value problem (1),(2).

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Necessary conditions for the extremum in non-smooth problems of variational calculus

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In the paper, we proposed an approach for studying strong and weak extremums in non-smooth vector problems of calculus of variation, namely, in classic variational problems with fixed ends and with a free right end, and also in a variational problem with higher derivatives. Two approaches are used: (a) Weierstrass variations in a new modified form; (b) Variation expressed by Legendre polynomials.

The theorem for strong and weak local extremums is proved in the considered problem. Let us consider a vector problem of calculus of variation in the form:

$$J\left(x\left(\cdot\right)\right)=\int_{t_0}^{t_1} L\left(t,x\left(t\right),x\left(t\right)\right)dt\rightarrow\min_{x(0)} \tag{1}$$

$$x\left(t_0\right)=x_0,\ x\left(t_1\right)=x_1,\ x_0,x_1\in R^n \tag{2}$$

where R^n is n-dimensional Euclidean space and x_0, x_1, t_0, t_1 points. The function $L\left(\cdot\right)$, $[t_0,t_1]\times R^n\times R^n\rightarrow R:=(-\infty;+\infty)$ called an integrant is assumed to be continuous by totality of variables. The sought-for function $x\left(\cdot\right):[t_0,t_1]=:I\rightarrow R^n$ is a piecewise smooth vector-function. We denote the set of such functions by $KC^1\left(;\,R^n\right)$.

Let $\overline{x}\left(\cdot\right)$ be some admissible function in the problem (1), (2) and $T_1\subseteq[t_0,\,t_1]$ be a set of continuity points of the function. We define the following function corresponding to the integrant $L\left(t,\,x,\,x\right)$ and the function $x\left(\cdot\right)$:

$$Q_1\left(t,\,\lambda,\,\xi,\,\overline{x}\left(\cdot\right)\right)=\lambda\left[L\left(t,\overline{x}\left(t\right),\tfrac{1}{x}\left(t\right)+\xi\right)-\overline{L}\left(t\right)\right]+\left(1-\lambda\right)\left[L\left(t,\overline{x}\left(t\right),\tfrac{1}{x}\left(t\right)+\tfrac{\lambda}{\lambda-1}\xi\right)-\overline{L}\left(t\right)\right],$$
$$\forall\left(t,\,\lambda,\,\xi\right)\in T_1x\times[0,1)\times R^n, \tag{3}$$

where $\overline{L}\left(t\right):=L\left(t,\,\overline{x}\left(t\right),\,\overline{x}\left(t\right)\right)$

Theorem 1. *Let the integrant $L\left(t,\,x,\,x\right):I\times R^n\times R^n\rightarrow R$ be continuous with respect to the totality of variables. Then:*

(a) if the function $\overline{x}\left(\cdot\right)$ is a strong local minimum in the problem (1), (2), the following inequality is fulfilled:

$$Q_1\left(t,\,\lambda,\,\xi,\,\overline{x}\left(\cdot\right)\right)\geq 0,\ \forall\left(t,\,\lambda,\,\xi\right)\in T_1\times[0,1)\times R^n \tag{4}$$

(b) if $\overline{x}\left(\cdot\right)$ is a weak local minimum in the problem (1), (2), then there exists a number $\delta>0$ such that the following inequality is fulfilled

$$Q_1\left(t,\,\lambda,\,\xi,\,\overline{x}\left(\cdot\right)\right)\geq 0,\ \forall\left(t,\,\lambda,\,\xi\right)\in T_1\times\left[0,\frac{1}{2}\right)\times B_\delta \tag{5}$$

where the function $Q_1(\overline{x}(\cdot))$ is determined by (3), the set $B_\delta(0)$ is a closed ball of radius δ centered at the point $0 \in R^n$.

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Strong kelley condition in the theory of singular optimal controls

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1. In this paper, we consider the following optimal control problem

$$S(u) = \varphi(x(t_1)) \to \min_u, \tag{1}$$

$$\dot{x}(t) = f(x(t),\ u(t), t),\quad t \in I := [t_0,\ t_1],\quad x(t_0) = x_0, \tag{2}$$

$$u(t) \in U \subset R^r,\quad t \in I, \tag{3}$$

where U is a given set in r- dimensional Euclidean space R^r; $R := (-\infty, \infty)$; $x \in R^n$ - state vector; $u \in U$- control vector and x_0, t_0, t_1 - fixed points, $\varphi(x) : R^n \to R$ and $f(x,\ u, t) : R^n \times R^r \times (a, b) \to R^n$ - given twice differentiable continuous function, where (a, b) is some interval, with $(a, b) \supset [t_0,\ t_1]$.

The function $u(\cdot)$ is called an admissible control, if $u(\cdot) \in KC(I,\ R^r)$ satisfies the condition (3), where $KC(I,\ R^r)$ is a class of piecewise continuous functions. An adimissible control $\overline{u}(\cdot)$, solution to the problem (1)-(3), is called an *optimal control*, and its corresponding solution $\overline{x}(\cdot)$ of (2) is called an *optimal trajectory.* A couple $(\overline{u}(\cdot),\ \overline{x}(\cdot))$ is called an *optimal process.*

As is known, in the study of problem (1)-(3) the concept of singular control was first introduced in 1959 by L.I. Rozonoer [1]. The problem of the optimality of singular controls in the classical sense was studied by H. Kelley and obtained a necessary condition based on the second variation of the problem (1)-(3) [2, 3]. To obtain this condition, H. Kelley introduced a new Weierstrass type variation. Later, the methods of studying singular controls proposed in [2-4] were essentially generalized in numerous works (see, for example, [4,5]).

In this paper, by proposing a variation based on research method introduced in [6], we obtain a necessary optimality condition for new classes of singular controls. The obtained condition is compared with Kelley's condition.

2. Let us formulate the obtained main result.

Definition 1. An admissible control $\overline{u}(\cdot)$ is called *singular* with respect to the system (2) in the interval $(\bar{t}_0,\ \bar{t}_1)$ for a point $(\lambda,\ v) \in (0,\ 1) \times U$, if the following relations hold

$$w(t) := \overline{u}(t) + \frac{\lambda}{\lambda - 1}(v - \overline{u}(t)) \in U,\ \forall t \in \left[\bar{t}_0,\ \bar{t}_1\right], \tag{4}$$

$$\lambda\ \Delta_v f(t) + (1 - \lambda\ )\,\Delta_{w(t)} f(t) = 0, \forall t \in (\bar{t}_0,\ \bar{t}_1), \tag{5}$$

where $\Delta_v f(t) := f(\overline{x}(t), v, t) - f(\overline{x}(t),\ \overline{u}(t), t)$ ($\Delta_{w(t)} f(t)$ is defined similarly).

It is clear that if the Pontryagin extremal $\overline{u}(\cdot)$ is singular in the sense of Definition 1, then it is also singular in the sense of Pontryagin, moreover, if $\overline{u}(t) \in intU,\ t \in I$, then the control $\overline{u}(\cdot)$ is also singular in a classical sense. Also note that if $f(x,u,t) = g(t,x) + G(t,x)u$, then the condition (5) is satisfied for all $(t,\ \lambda,\ u) \in I \times (0,1) \times U$.

Theorem 1. *Let an admissible control $\overline{u}(t)$, $t \in I$ be singular with respect to the system (2) in the interval $(\bar{t}_0,\ \bar{t}_1) \subset I$ for a point $(\lambda,\ v) \in (0,1) \times U$. In addition, let the functions $\overline{u}(t)$ and $\dot{\overline{u}}(t)$ be continuous at the points of the interval $(\bar{t}_0,\ \bar{t}_1)$. Then, for the optimality of admissible control $\overline{u}(\cdot)$ it is necessary that the following inequality holds*

$$\lambda\Delta_v f^T(t)\,[\lambda H_{xx}(v,t) + (1-\lambda)\,H_{xx}(w(t),\ t)]\Delta_v f(t) +$$

$$+\lambda^2\Delta_v H_x^T(t)\,[f_x(v,t)\,\Delta_v f(t) + \frac{d}{dt}\Delta_v f(t)] +$$

$$+\Delta_{w(t)}H_x^T(t)[(1-\lambda)^2(2f_x(w(t),t)\Delta_v f(t) - \frac{d}{dt}\Delta_v f(t)) +$$

$$+3\lambda(1-\lambda)f_x(v,t)\Delta_v f(t)] + [(1-\lambda)(1+2\lambda)\frac{d}{dt}\left(\Delta_{w(t)}H_x(t)\right) +$$

$$+2\lambda^2\frac{d}{dt}(\Delta_v \mathrm{H}_x(t))]\Delta_v f(t) \le 0,\ \forall t \in (\bar{t}_0,\ \bar{t}_1). \qquad (6)$$

Here $H(\psi,x,u,t) = \psi^T f(x,u,t), \dot{\overline{\psi}}(t) = -H_x(t), \overline{\psi}(t_1) = -\varphi_x(\overline{x}(t_1))$,
$\Delta_{w(t)}H_x(t) = H_x\left(\overline{\psi}(t),\ \overline{x}(t), w(t),\ t\right) - H_x(\overline{\psi}(t),\ \overline{x}(t),\ \overline{u}(t), t)$,
$H_{xx}(v,t) = H_{xx}\left(\overline{\psi}(t),\ \overline{x}(t),\ v,\ t\right)$ $(\Delta_v H_x(t),\ H_{xx}(w(t),t), f_x(v,t), f_x(w(t),t)$ *are defined similarly, where* $w(t) = \overline{u}(t) + \frac{\lambda}{\lambda-1}(v - \overline{u}(t)))$.

Theorem 2. *Let an admissible control $\overline{u}(\cdot)$ is a singular with respect to the system (2) in the interval $(\overline{t_0},\overline{t_1})$ for the points $v \in U$ and for all $\lambda \in (0,\lambda_0)$, where $\lambda_0 \in (0,1)$. In addition, let $\overline{u}(t) \in intU$, $t \in [\overline{t}_0,\overline{t}_1]$ and functions $\overline{u}(t)$ and $\dot{\overline{u}}(t)$ be continuous in $(\overline{t_0},\overline{t_1})$. Then, for the optimality of admissible control $\overline{u}(\cdot)$ it is necessary that the following inequality holds*

$$-\Delta_v f^T(t)\,H_{xx}(t)\,\Delta_v f(t) + 2\Delta_v H_x^T(t)[f_x(t)\Delta_v f(t) - \frac{d}{dt}\Delta_v f(t)] +$$

$$+\frac{d}{dt}[\Delta_v H_x^T(t)\Delta_v f(t)] \ge 0,\ v \in U,\ \forall t \in (\overline{t_0},\overline{t_1}). \qquad (7)$$

Note that from condition (7) it is easy to obtain Kelley's conditions [3]. An example is given showing that, under the conditions of Theorem 2, the necessary optimality condition (7) is stronger than the Kelley conditions. Therefore, the necessary optimality condition (7) can be called the strong Kelley condition in the theory of singular optimal controls.

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Integral representation of functions in Sobolev's anisotropic space with a dominant mixed derivative

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Various integral representations for functions of several variables from Sobolev spaces with dominant mixed derivatives have been developed by many mathematicians and have received new interesting and important applications [1-6]. In this paper, for the first time, one integral representation is found for functions of three variables in one anisotropic Sobolev space with a dominant mixed derivative. The result obtained is used in the study of three-dimensional non-classical boundary value problems for equations with a dominant mixed derivative $D_1^2 D_2\, u$ with L_p coefficients in the space of S. L. Sobolev $W_p^{(2,1)}(G)$.

Let $G_\tau = (x_\tau^0, h_\tau), (\tau = 1, 2)$, $G = G_1 \times G_2$, $W_p^{(2,1)}(G) = \{u(x) \in L_p(G)/D_1^{i_1}$, $D_2^{i_2}\, u \in L_p(G);\;\; i_1 = 0, 1, 2; i_2 = 0, 1; \}$, $1 \le p \le \infty$, where $x = (x_1, x_2) \in G, D_\tau = \partial/\partial x_\tau$, $(\tau = 1,\, 2)$ operator of generalized differentiation in the sense of S. L. Sobolev. We define the norm in it by the equality

$$\|u\|_{W_p^{(2,1)}(G)} = \sum_{i_1=0}^{2} \sum_{i_2=0}^{1} \left\| D_1^{i_1} D_2^{i_2} u \right\|_{L_p(G)}.$$

Theorem. If $u(x) \in W_p^{(2,1)}(G)$, then the function $u(x)$ can be represented as

$$u(x) = u\left(\frac{x_1^0 + h_1}{2}, \frac{x_2^0 + h_2}{2}\right) + \left(x_1 - \frac{x_1^0 + h_1}{2}\right) D_1 u\left(\frac{x_1^0 + h_1}{2}, \frac{x_2^0 + h_2}{2}\right) +$$

$$+ \int_{\frac{x_1^0+h_1}{2}}^{x_1} (x_1 - \alpha_1) D_1^2 u\left(\alpha_1, \frac{x_2^0 + h_2}{2}\right) d\alpha_1 + \int_{\frac{x_2^0+h_2}{2}}^{x_2} D_2 u\left(\frac{x_1^0 + h_1}{2}, \alpha_2\right) d\alpha_2 +$$

$$+ \left(x_1 - \frac{x_1^0 + h_1}{2}\right) \int_{\frac{x_2^0+h_2}{2}}^{x_2} D_1 D_2 u\left(\frac{x_1^0 + h_1}{2}, \alpha_2\right) d\alpha_2 +$$

$$+ \int_{\frac{x_1^0+h_1}{2}}^{x_1} \int_{\frac{x_2^0+h_2}{2}}^{x_2} (x_1 - \alpha_1) D_1^2 D_2 u(\alpha_1, \alpha_2) d\alpha_1 d\alpha_2.$$

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On solvability of a mixed problem for one form of non-classical equations

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In a rectangular domain of variables t and x by the contour method we study the solvability of a simplest mixed problem for a class of equations with complex-valued coefficients. These equations are characterized by the following features :

1. They behave as parabolic equations despite the fact that over time can to from parabolic type to anti-parabolic type to Schrödinger or even to anti-parabolic type.

2. For the equations of appropriate spectral problems, the arguments of the roots of J. Birkhoff characteristical polynomials are not constant.

We study the solvability of the mixed problem

$$M\left(t,\frac{\partial}{\partial t}\right)U = L\left(x,\frac{\partial}{\partial x}\right)U, 0<t<T,\ \ 0<x<1 \tag{1}$$

$$U(0,x)=\varphi(x) \tag{2}$$

$$U(t,0)=U(t,1)=0 \tag{3}$$

where $M\left(t,\frac{\partial}{\partial t}\right)=\frac{1}{P(t)}\frac{\partial}{\partial t}$, $L\left(x,\frac{\partial}{\partial x}\right)=\frac{1}{(x+b)^2}\cdot\frac{\partial^2}{\partial x^2}, b=b_1+ib_2$, $P\left(t\right)=p_1\left(t\right)+ip_2\left(t\right)$, are complex-valued functions, $p_j\left(t\right)\in C[0,\,1]\ \ (j=1,\,2)$, $P_1\left(t\right)\neq 0$, $\varphi\left(x\right)$ is a given $U\left(x\right)$ is a sought for function .

Terminal solvability conditions will be

$$1^0.\int_0^t p_1\left(\tau\right)d\tau\ >0,\ b_1>0\,,\,b_2>0,;$$

2^0. $Reb^2+\omega\left(T\right)Imb^2>0$, if $Im\left[\overline{p}\cdot\int_0^t p\left(\tau\right)d\tau\right]\geq 0$ and

$Reb^2+\omega\left(T\right)Imb^2<0$ if $Im\left[\overline{p}\cdot\int_0^t p\left(\tau\right)d\tau\right]<0$ where $\omega\left(t\right)=\int\limits_0^t p_2\left(\tau\right)d\tau\cdot\left(\int_0^t p_1\left(\tau\right)d\tau\right)^{-1}$

$$3^0.\varphi(x)\in C^2[0,\ 1],\varphi(0)=\varphi(1)=0.$$

Note that the condition 1^0 allows one to go beyond the I.G. Petrovsky parabolicity (or well-posedness) of equation (1). Obviously, subject to the condition 2^0, the equation (1) is I.G. Petrovsky parabolic to if and only if

$$ReP_1\left(t\right)>0\,,\,0\leq t\leq T\,,$$

while under the condition 1^0, Re $P_1(t)$ can be zero or negative in some part of $(0, T]$.

The following theorem is valid.

Theorem. *Let conditions*1^0, $2^0, 3^0$ *be fulfilled. Then the problem (1)-(3) has a classic solution* $U(t, x) \in C^{1,2}((0;T] \times [0;1]) \bigcap C([0;T] \times [0;1])$ *representable by the following formula for* $t > 0$

$$U(t, x) = \frac{1}{\pi i} \int \lambda 5^{\lambda^2 \int_0^t P(\tau)\tau} \cdot y(x, \lambda) \, d\lambda$$

where

$$\Gamma = \bigcup_{j=1}^{3} \Gamma_j$$

$$\Gamma_j = \{\lambda : \lambda = r(1 + p_j), r \geq R\} \ (j = 1, 2)$$

$$\Gamma_3 = \{\lambda : \lambda = R(1 + i\eta), p_1 \leq \eta \leq p_2\} \ ,$$

$$y(x, \lambda) = \int_0^1 G(x, \xi, \lambda) (\xi + b)^2 \varphi(\xi) \, d\xi$$

$$p_j = K_j(t_j) + (-1)^j \delta, \ K_j(t_j) = -\omega(t) + (-1)^j \sqrt{\omega^2(t) + 1}, \ (j = 1, 2)$$

$$\omega(t) = \int_0^t p_2(\tau) \, d\tau \cdot \left(\int_0^t p_1(\tau) \, d\tau \right)^{-1}$$

$t_1 = 0, \ t_2 = T$ *if* $Im \left[\overline{p} \cdot \int_0^t p(\tau) \, d\tau \right] \geq 0$ *and* $t_1 = T, \ t_2 = 0$ *if* $Im \left[\overline{p} \cdot \int_0^t p(\tau) \, d\tau \right] < 0$, R *is a rather large,* δ *is a rather small positive value.*

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Longitudinal shift relaxed rectified through crack

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Formulation of the problem. Let S_k^t and S_k^a-be the regions of the cell structure occupied by the circular coating of the $A^a e-$th fiber and the fiber itself, the center of which is located at the point $a_k + P$,

$$(1 + G_t \backslash G_s)\, \varphi_k\,(\tau_k) + (1 - G_t \backslash G_s)\, \overline{\varphi_k\,(\tau_k)} = 2 f_s\,(\tau_k)\,(k = 1, 2, \ldots, n)\,. \quad (1)$$

resulting from ideal contact conditions. The boundary conditions on the banks of the cracks are:

$$\varphi'_s\,(t) - \overline{\varphi'_s\,(t)} = 0, \varphi'_s\,(t_1) - \overline{\varphi'_s\,(t_1)} = 0, \varphi'_b\,(t) - \overline{\varphi'_b\,(t)} = 0. \quad (2)$$

The method of solving the boundary value problem. To determine arbitrary constants, boundary conditions are considered where instead of the functions W_a, W_s, W_t and the corresponding stresses are introduced in the Fourier series.

After finding the value P_ν^0, the stress intensity factor K_{III} is determined on the basis of relations:

$$K_{III}^a = \sqrt{\frac{\pi l \left(1 - \lambda_1^2\right)}{\lambda_1}}\, \frac{1}{2n} \sum_{\nu=1}^{n} (-1)^{\nu+n} P_\nu^0 \; tg \frac{\theta_\nu}{2}; K_{III}^l = \sqrt{\pi l \left(1 - \lambda_1^2\right)}\, \frac{1}{2n} \sum_{\nu=1}^{n} (-1)^{\nu} P_\nu^o \frac{\theta_\nu}{2}. \quad (3)$$

Decision analysis. Based on the results in plot the dependence of the critical (limiting) load $\tau_* = \tau_y^\infty \sqrt{\omega_1}/K_{III\ c}$ for both crack tips on its length $l_* = a - l$ for some values of the radius of the hole $\lambda = 0.2;\ 0.3;\ 0.4;\ 0.5;\ 0.6$.

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On the generalized Hardy inequality and the best constant

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In this abstract, we show an elementary approach to prove the Hardy-Sobolev type

$$\|dist(x,\partial\Omega)^{\alpha-1}\|_{L^p(\Omega)} \leq C_{p,\alpha}\|dist(x,\partial\Omega)^{\alpha}\nabla f\|_{L^p(\Omega)}$$

for any $f \in Lip_0(\Omega)$ and $-\infty < \alpha < (p-1)p, 1 < p < \infty$, the $\Omega \subset R^n$ is a bounded convex domain. $dist(x,\partial\Omega)$ is a distance from the point $x \in \Omega$ to boundary $\partial\Omega$. For $\alpha = 0$ this inequality and its generalisations are well-studied in the literature (see e.g. [2]) and the best constant $C_{p,0} = p'(p-1)$ is known not achieved, i.e. no a function $u_0(x)$ exists for which this inequality turns into equality (see, the Monograph [1]). To prove the cited inequality we use the next one-dimensional auxiliary assertion.

Lemma. *Let* $1 < p < \infty, -\infty < \alpha < \frac{1}{p^{`}}$, $n > 1$. *Let* $u(x)$ *be an absolutely continuous function on finite interval* $(0,l)$ *such that* $\lim\limits_{x\to+0} u(x) = 0$ *and* $\lim\limits_{x\to 1} u(x) = 0$. *Then it holds the inequality*

$$\|x^{(n-1)/p}, \frac{u(x)}{d(x)^{1-\alpha}}\|_{L^p(0,l)} \leq \frac{p}{p-1-\alpha p}\|d(x)^{\alpha}x^{(n-1)/p}, \frac{du}{dx}\|_{L^p(0,l)}$$

where $d(x) = min(x, l-x)$.

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Intelligent information system of monitoring, diagnosis and prognosis of diseases in urgent therapy using carbon monoxide poisoning as an example

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According to statistical data, with the development of oil, chemical, gas industries cases of poisoning caused by toxic substances employed in these branches have become more frequent recently. While it is quite easy to diagnose carbon monoxide poisoning unmistakably in everyday life this procedure may become just as complicated in production conditions where a large quantity of toxic substances are present simultaneously and among the said substances there are ones causing almost similar symptoms of affection [1]. This fact is particularly undesirable when the first and urgent aid are needed for which time and absence of laboratory data are the main factors. At the pre-hospital stage, it is important to eliminate generated pathological syndromes and to take measures for active detoxication of organism. If one manages to reveal a poisoning substance, exactly it becomes vital to apply antidote therapy (removal of a poison from organism) which is specific for each toxin.

$$y_i = \sum_{i=m,k,l,\ldots}^{38} x_i^{+} \sum_{i=t,s,p,\ldots}^{38} x_i^{-} + \alpha \left(\sum_{i=j,r,v,\ldots}^{38} x_i^{h/y} \right)$$

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Ordinary self-adjoint differential operators and integral representation of the sums of some series

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In papers [1] and [2], an original method was proposed and developed based on the application of the spectral theory of ordinary differential operators generated by symmetric differential expressions with constant coefficients and self-adjoint boundary conditions in the space $\mathcal{L}^2[a,b]$ to obtain new integral representations of the sums some numerical series and special functions (polylogarithmic function, Euler digamma functions, values of the Riemann zeta function and related functions at natural points, etc.) in the form of definite integrals of elementary (trigonometric) functions and, in particular, to the derivation of formulas for the sums of some convergent numerical series.

Using these integral representations, on the one hand, it is possible to uniformly prove both formulas that have already become classical and formulas recently obtained by other authors, and on the other hand, numerous new ones can be obtained.

The report is devoted to this range of issues.

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On approximation of $|x|$ by trigomometric polynomials

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Classical results by S. N. Bernstein [1] and I. I. Ibragimov [2] about $|x|$ and $x \ln x$ play a crucial role in the approximation by algebraic polynomials. It turns out, their analogs are needed in the approximation by trigonometric polynomials, at least in the Shape Preserving Approximation. So, D. Leviatan and the authors [3] proved

Theorem 1. *Let $0 < b \leq \pi$. For each polynomial*

$$T_n(x) = a_0 + \sum_{k=1}^{n} \cos kx$$

we have

$$\max_{x\in[-b,b]} \mid |x| - T_n(x) \mid > \frac{c}{n},$$

where $c = c(b) > 0$ depends only on b.

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On smooth solutions of one class of fourth order operator-differential equations with multiple characteristics

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The paper considers a fourth-order operator-differential equation of the form

$$\left(-\frac{d}{\mathrm{dt}}+A\right)^{4-k}\left(\frac{d}{\mathrm{dt}}+A\right)^{k}u(t)+$$

$$+\sum_{j=1}^{3}A_j u^{(4-j)}(t)=f(t),\quad t\in R=(-\infty,+\infty), \tag{1}$$

where A be a self-adjoint positive definite operator in a separable Hilbert space H, $A_j,\ j=1,2,3$, are linear, generally speaking, unbounded operators in H, $f(t)\in W_2^1(R;H)$, $u(t)\in W_2^5(R;H)$, $k=1$. Here by $W_2^m(R;H)$, for integers $m\geq 1$, we mean the Hilbert space (see [1])

$$W_2^m(R;H)=\left\{u(t):\frac{d^m u(t)}{dt^m}\in L_2(R;H), A^m u(t)\in L_2(R;H)\right\}$$

with the norm

$$\|u\|_{W_2^m(R;H)}=\left(\left\|\frac{d^m u}{dt^m}\right\|^2_{L_2(R;H)}+\|A^m u\|^2_{L_2(R;H)}\right)^{1/2},$$

where $L_2(R;H)$ denotes the Hilbert space of vector-functions $f(t)$, defined in R, with values in H, and for which

$$\|f\|_{L_2(R;H)}=\left(\int_{-\infty}^{+\infty}\|f(t)\|_H^2\,\mathrm{dt}\right)^{1/2}<+\infty.$$

Derivatives are understood in the sense of distribution theory (see [1]).

We denote by H_θ the scale of Hilbert spaces generated by the operator A, that is,

$$H_\theta=Dom\left(A^\theta\right),\ \theta\geq 0,(x,y)_\theta=(A^\theta x,A^\theta y),x,y\in Dom(A^\theta).$$

Traditionally, $L(X,Y)$ is understood as a set of linear bounded operators acting from a Hilbert space X to another Hilbert space Y.

The following theorem is proved in the paper.

Theorem. *Let A be a self-adjoint positive definite operator in H and the operators $A_j \in L(H_j, H) \cap L(H_{j+1}, H_1)$, $j = 1, 2, 3,$ and the following inequality is true*

$$\sum_{j=1}^{3} \max\left\{\left\|A_{4-j}A^{-(4-j)}\right\|_{H\to H}, \left\|AA_{4-j}A^{-(5-j)}\right\|_{H\to H}\right\} n_j < 1,$$

where $n_j = \frac{1}{16} j^{j/2}(4-j)^{(4-j)/2}$, $j = 1, 2, 3$. Then, for any $f(t) \in W_2^1(R; H)$, equation *(1) has a unique solution $u(t) \in W_2^5(R; H)$.*

Note that the case $k = 3$ was studied in [2].

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Boundedness of anisotropic singular operator in anisotropic generalized Morrey spaces

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In this paper we give the conditions on the pair (φ_1, φ_2) which ensures the boundedness of the anisotropic singular integral operators from one generalized Morrey space $\mathcal{M}_{p,\varphi_1}$ to another $\mathcal{M}_{p,\varphi_2}$, $1 < p < \infty$, and from the space $\mathcal{M}_{1,\varphi_1}$ to the weak space $W\mathcal{M}_{1,\varphi_2}$.

We consider the following anisotropic singular integral

$$Tf(x) = \text{p.v.} \int_{\mathbb{R}^n} k(x; x-y) f(y) dy$$

with a variable Calderón-Zygmund type kernel $k(x, \xi)$, $x \in \mathbb{R}^n$, $\xi \in \mathbb{R}^n \setminus \{0\}$, satisfying a mixed homogeneity condition $k(x; A_t\xi) = t^{-\gamma} k(x; \xi),\ t > 0$.

Theorem. *Let $p \in [1, \infty)$ and (φ_1, φ_2) satisfy the condition*

$$\left(\int_t^1 \left(\frac{ess \inf\limits_{r<s<1} \varphi_1(x,s)}{r^\gamma} \right)^{\frac{1}{p}} \frac{dr}{r} \right)^p \le C \, \frac{\varphi_2(x,t)}{t^\gamma}.$$

Then the anisotropic singular integral Tf exists for a.e. $x \in \mathbb{R}^n$; and for $p > 1$ the operator T is bounded from $\mathcal{M}_{p,\varphi_1}(\mathbb{R}^n)$ to $\mathcal{M}_{p,\varphi_2}(\mathbb{R}^n)$, and for $p = 1$ the operator T is bounded from $\mathcal{M}_{1,\varphi_1}(\mathbb{R}^n)$ to $W\mathcal{M}_{1,\varphi_2}(\mathbb{R}^n)$. Moreover, for $p > 1$

$$\|Tf\|_{\mathcal{M}_{p,\varphi_2}} \lesssim \|f\|_{\mathcal{M}_{p,\varphi_1}},$$

and for $p = 1$

$$\|Tf\|_{W\mathcal{M}_{1,\varphi_2}} \lesssim \|f\|_{\mathcal{M}_{1,\varphi_1}}.$$

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General solution of homogeneous Riemann problem in Hardy-Morrey classes

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To establish the basicity of the exponential system

$$E_\beta \equiv \left\{ e^{i(nt+\,\beta|t|\,sign\;n)} \right\}_{n\in Z}$$

for Morrey-type spaces $M^{p,\alpha}$, we will use the method of Riemann boundary value problems developed by B.T. Bilalov (see, e.g., [1, 2, 3, 4, 5, 6, 7, 8, 9]). Consider the following homogeneous Riemann problem:

$$\left. \begin{array}{l} F^+(\tau) - G(\tau) F^-(\tau) = 0, \qquad \tau \in \gamma, \\ F^+(\cdot) \in H_+^{p,\alpha}; F^-(\cdot) \in_m H_-^{p,\alpha}, \end{array} \right\} \qquad (2)$$

$1 < p < +\infty,\ \ 0 < \alpha \leq 1$, where $G\left(e^{it}\right) = \left|G\left(e^{it}\right)\right| e^{i\theta(t)},\ \ t \in [-\pi, \pi]$ is the coefficient of the problem. By the solution of the problem (2) we mean a pair of functions $(F^+; F^-) \in H_+^{p,\alpha} \times_m H_-^{p,\alpha}$ such that the non-tangential boundary values $F^+(\tau)$ inside ω and $F^-(\tau)$ outside ω satisfy the relation (2) a.e. on γ. Introduce the following piecewise analytic functions on the complex plane cut by γ:

$$X_1(z) \equiv \exp\left\{ \frac{1}{4\pi} \int_{-\pi}^{\pi} \ln\left|G\left(e^{it}\right)\right| \frac{e^{it}+z}{e^{it}-z} dt \right\},$$

$$X_2(z) \equiv \exp\left\{ \frac{i}{4\pi} \int_{-\pi}^{\pi} \theta(t) \frac{e^{it}+z}{e^{it}-z} dt \right\},\ z \notin \gamma.$$

Let

$$Z_k(z) = \left\{ \begin{array}{ll} X_k(z), & |z| < 1, \\ (X_k(z))^{-1}, & |z| > 1, \end{array} \right.$$

and $Z(z) = Z_1(z) Z_2(z),\ z \notin \gamma$.

Function $Z(\cdot)$ will be called a canonical solution of homogeneous problem.

Regarding the coefficient $G(\cdot)$ of the problem (2), we will assume that the following conditions hold:

i) $G^{\pm 1}(\cdot) \in L_\infty(-\pi, \pi)$;

ii) $\theta(t) = \arg G\left(e^{it}\right)$ is a piecewise Hölder function on $[-\pi, \pi]$, and let $h_k = \theta(s_k+0) - \theta(s_k-0),\ k = \overline{1,r}-$ be the jumps of this function at the points of discontinuity $\{s_k\}_1^r$: $-\pi < s_1 < .... < s_r < \pi$.

B.T. Bilalov [10] proved the following theorem.

Theorem 1. *Let the coefficient* $G(\cdot)$ *of the problem* (2) *satisfy the conditions* i), ii) *and the jumps* $\{h_k\}_0^r$ *of the function* $\theta(t) = \arg G(e^{it})$ *on* $[-\pi, \pi]$, *where* $h_0 = \theta(-\pi) - \theta(\pi)$, *satisfy the inequalities*

$$-1 + \frac{\alpha}{p} < \frac{h_k}{2\pi} \leq \frac{\alpha}{p}, \quad k = \overline{0, r}.$$

Then:

α) *for* $m \geq 0$ *the problem* (2) *has a general solution of the form*

$$F(z) \equiv Z(z) P_k(z), \tag{3}$$

where $Z(z)$ *is a canonical solution of this problem, and* $P_k(z)$ *is an arbitrary polynomial of degree* $k \leq m$;

β) *for* $m < 0$ *the problem* (2) *has only a trivial solution.*

Using this theorem, the following theorem is proved.

Theorem 2. *Let the coefficient* $G(\cdot)$ *satisfy the conditions* i), ii) *and the jumps* $\{h_k\}_0^r$ *of the function* $\theta(\cdot)$ *satisfy the inequalities*

$$-1 + \frac{\alpha}{p} < \frac{h_k}{2\pi} < \frac{\alpha}{p}, \qquad k = \overline{0, r}.$$

Then:

α) *for* $m \geq 0$ *the problem* (2) *has a general solution of the form* (3) *in the Morrey-Hardy classes* $MH_+^{p;\alpha} \times_m MH_-^{p;\alpha}$, $1 < p < +\infty$, $0 < \alpha \leq 1$;

β) *for* $m < 0$ *the problem* (2) *has only a trivial solution.*

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On a dispersion problem with a singular potential of measure type

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This paper is devoted to the spectral analysis of the Schrödinger operator in E_3 with a complex-valued potential, which is a generalized function of a certain form, corresponding to a singular interaction characterized by a measure-type potential.

Let $q(x)$ a generalized function that is generated with a complex measure σ :

$$(q(x), \varphi) = \int_{E_3} \varphi(x)\, d\sigma(x).$$

Here σ is a complex-valued countably additive measure defined on the σ -algebra of subsets from E_3 and satisfies the conditions:

a) $\int\limits_{E_3} e^{\alpha|x|} |d\sigma(x)| < \infty, \alpha > 0;$

b) the $\int\limits_{E_3} \frac{|d\sigma(y)|}{|x-y|}$ exists and is continuous with respect to $x \in E_3$;

c) for $\mu \to +\infty$, $\int\limits_{E_3} \frac{e^{-\mu|x-y|}}{|x-y|} |d\sigma(y)| \to 0$ uniformly in $x \in E_3$.

Let us define operator L in $L_2(E_3)$:

$$D(L) = \{\psi \in L_2 | \psi \in C \cap L_\infty - \Delta\psi + q\psi \in L_2\},\ L\psi = -\Delta\psi + q\psi \in L_2$$

Denote by $H = L_2(E_3, \alpha)$ the Hilbert space of complex-valued measurable functions $f(x)$ with the norm

$$\|f\|_H^2 = \int_{E_3} |f(x)|^2\, d\mu(x),\ d\mu(x) = e^{-\alpha|x|}dx$$

Definition. Solution $\psi(x, k, \omega)$ from H of the integral equation

$$\psi(x, k, \omega) = e^{ik(x,\omega)} - \int_{E_3} \frac{e^{ik|x-y|}}{4\pi|x-y|} \psi(y, k, \omega)\, d\sigma(y) \quad (1)$$

is called the solution of the scattering theory problem (s.t.p.) for the equation

$$L\psi = \lambda\psi,$$

where $k^2 = \lambda$ ω arbitrary unit vector from E_3.

Let us rewrite equation (1) in the form

$$\psi(x,k,\omega) = e^{ik(x,\omega)} + T_k\psi(x,k,\omega), \tag{2}$$

where $T_k\psi(x,k,\omega) = \int\limits_{E_3} G(x,y,k)\,\psi(y,k,\omega)\,d\sigma(y)$, $G(x,y,k) = \frac{e^{ik|x-y|}}{4\pi|x-y|}$

Let us consider the homogeneous equation

$$f(x) = -\int\limits_{E_3} G(x,y,k)\,f(y)\,d\sigma(y), \tag{3}$$

corresponding to the equation of the (s.t.p.) (1). The following statements have been proved:

Theorem 1. *Let $Imk > -\alpha/2$ and $f(x) \in C \cap L_\infty$ is a solution of the homogeneous equation (3). Then $f(x) \in H$ and $Lf = \lambda f$ in the sense of generalized functions, where $k^2 = \lambda$.*

Theorem 2. *Under conditions a), b), c) the operator has only a finite number of eigenvalues and a finite number of spectral singularities.*

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A note on some restricted inequalities for the iterated Hardy-type operator involving suprema

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In this talk, we present a characterization of the inequality

$$\left(\int_0^\infty \left(\int_0^x \left[T_{u,b} f^*\right]^r\right)^{\frac{q}{r}} w(x)\,dx\right)^{\frac{1}{q}} \le C \left(\int_0^\infty \left(\int_0^x [f^*]^p\right)^{\frac{m}{p}} v(x)\,dx\right)^{\frac{1}{m}}$$

for $1 < m < p \le r < q < \infty$ or $1 < m \le r < \min\{p,q\} < \infty$, where w and v are weight functions on $(0,\infty)$. Here f^* is the non-increasing rearrangement of a measurable function f defined on $\mathbb{R}^n$ and $T_{u,b}$ is the iterated Hardy-type operator involving suprema, which is defined for a measurable non-negative function f on $(0,\infty)$ by

$$(T_{u,b}g)(t) := \sup_{t \le \tau < \infty} \frac{u(\tau)}{B(\tau)} \int_0^\tau g(s)b(s)\,ds, \qquad t \in (0,\infty)$$

where u and b are two weight functions on $(0,\infty)$ such that u is continuous on $(0,\infty)$ and the function $B(t) := \int_0^t b(s)\,ds$ satisfies $0 < B(t) < \infty$ for every $t \in (0,\infty)$.

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New equivalence theorems for monotone quasilinear operators

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In this talk, we present new equivalence theorems for the boundedness of the composition of a quasilinear operator T with the Hardy and Copson operators in weighted Lebesgue spaces. An application of the obtained results is illustrated in the case of weighted Hardy-type and weighted iterated Hardy-type inequalities.

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Elastico-plastic torsional deformation of a hardening beam

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A problem of elastic-plastic deformation of a circular cross section beam of radius R in its torsion by the torque $M$is studied. Hypothesis of plane sections and assumption that when the beam is under torsion, radial fibers remain rectilinear with invariable angles between them are used. In the cylindrical system of coordinates(r, φ, z) the displacement vector components $\vec{u}\,(u_r, u_\varphi, u_z)$ will be $u_r = u_z = 0$, $u_\varphi = 2\theta\, r\, z$, where θ is a relative twisting angle. The strain tensor components are the followings: $\varepsilon_r = \varepsilon_\varphi = \varepsilon_z = \varepsilon_{r\varphi} = \varepsilon_{rz} = 0,\ \varepsilon_{z\varphi} = \theta\, r$. It is admitted that the material of the beam has hardening. Therewith, the lateral area of the beam or $0 \le r \le R$ is divided into the area of elastic $0 \le r \le r_s$ and plastic $r_s \le r \le R$ deformations. The Hooke law is valid in the elastic area. A.A. Ilyushin's theory of small elastico-plastic deformations hold. in the plastic area [1]. In the given problem it remains only one equation from six determining relations: $\sigma_{z\varphi}^{(e)} = 2C\varepsilon_{z\varphi}$, in the domain $0 \le r \le r_s$; $\frac{\sigma'_{z\varphi}}{2G_0} = \varphi\left(\frac{2\varepsilon_{z\varphi}}{\sqrt{3}}\right)\varepsilon_{z\varphi}$, in the domain $r_s \le r \le R$, where G is a shift modulus of the beam material, φ is a plastic hardening function. The power function $\varphi\,(v) = Bv^\beta$ was used for the approximation φ.

Here B and β are the constants of the beam material. Elastic deformation in the beam is

$$\sigma_{z\varphi}^{(e)} = \left(\frac{2}{\pi R^3}\right)M.$$

Plastic deformation is:

$$\sigma_{z\varphi}^{(e)} = \left[M\,(\beta+4)\,(r/R)^{1+\beta}\right]\left(2\pi R^3\right).$$

First plastic deformations appear at the points of the surface of the beam at the torque $M_s = \frac{\pi R^3}{2\sqrt{3}}\sigma_s$, where σ_s is a yield point of the beam material in expansion.

The boundary (r_s) of elastic and plastic areas are determined from the condition of equality of stress $\sigma_{z\varphi}$ and displacement u_φ for $r = r_s$ on the boundary of these areas.

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On relations between some orthogonal polynomials

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In this paper, we establish relations between orthogonal Hermite $H_n(x)$, Laguerre $L_n^{\alpha}(x)$, Jacobi $P_n^{(\alpha,\ \beta)}(x)$, pseudo-Jacobi $P_n(x;;\nu,N)$, Meixner-Pollaczek $P_n^{\nu}(x;;\varphi)$, Meixner $M(x;;\beta,c)$ and continuous dual Hahn $S_n\left(x^2;;a,b,c\right)$ polynomials.

Theorem 1. *The Hermite polynomials $H_n\left(x+\frac{a-b}{2}\right)$ are obtained from the Jacobi polynomials $P_n^{(\alpha,\ \beta)}(x)$ according to the following relationship:*

$$\lim_{\nu\to\infty} 2^n n!\nu^{-n} P_n^{\left(\nu^2+a\nu+c_1\ ,\nu^2+b\nu+c_2\right)}\left(\frac{x}{\nu}\right) = H_n\left(x+\frac{a-b}{2}\right).$$

Theorem 2. *The Laguerre polynomials $L_n^{a-\frac{1}{2}}\left(x^2\right)$ are obtained from the continuous dual Hahn polynomials $S_n(x^2;;a,b,c)$ according to the following relation:*

$$\lim_{\nu\to\infty}\frac{1}{n!\nu^n}S_n(x^2\nu;;a,\nu,\frac{1}{2}) = L_n^{a-\frac{1}{2}}\left(x^2\right).$$

Theorem 3. *The Hermite polynomials $(-1)^n H_n(x)$ are obtained from the Meixner polynomials $M_n(x;;\beta,c)$ according to the following relation:*

$$\lim_{\nu\to\infty}(2\nu)^{n/2}M_n\left(\frac{\nu+\sqrt{2\nu}x}{1-c};;\frac{\nu}{c},c\right) = (-1)^n H_n(x).$$

Theorem 4. *The Meixner-Pollaczek polynomials $P_n^{\nu}(x;;\varphi)$ and Laguerre $L_n^{\alpha}(x)$ are interconnected by the following integral relations:*

$$\int_0^{\infty} e^{-\frac{1}{2}(1-i\mathrm{ctg}\varphi)t}t^{\nu+ix-1}L_n^{2\nu-1}(t)\,dt = (2\sin\varphi)^{\nu+ix}e^{\left(\frac{\pi}{2}-\varphi\right)(i\nu-x)}\tilde{A}(\nu+ix)\,e^{-in\varphi}P_n^{\nu}(x;;\varphi),$$

$$\int_{-\infty}^{\infty} t^{-ix}\tilde{A}(\nu+ix)e^{x\left(\varphi-\frac{\pi}{2}\right)}P_n^{\nu}(x;;\varphi)\,dx = 2\pi e^{in\varphi}e^{i\left(\varphi-\frac{\pi}{2}\right)\nu}\exp\left(ite^{i\varphi}\right)\ t^{\nu}L_n^{2\nu-1}(2t\sin\varphi),$$

where $t>0,\nu>0$, $n=0,\ 1,\ 2,\ \ 3,\ldots$

Consequence. *These integral formulas can be rewritten in "local" form:*

$$L_n^{2\nu-1}\left(e^{-i\partial_x}\right)(2\sin\varphi)^{ix}e^{x\left(\varphi-\frac{\pi}{2}\right)}\tilde{A}\left(\nu+ix\right)=(2\sin\varphi)^{ix}e^{x\left(\varphi-\frac{\pi}{2}\right)}\tilde{A}\left(\nu+ix\right)e^{-in\varphi}P_n^{\nu}\left(x;;\varphi\right),$$

$$P_n^{\nu}\left(it\partial_t;;\varphi\right)t^{\nu}e^{ite^{i\varphi}}= t^{\nu}\exp\left(ite^{i\varphi}\right)\ e^{in\varphi}L_n^{2\nu-1}(2t\sin\varphi).$$

Theorem 5. *The following summation formula holds for Hermite polynomials*

$$n!\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{\left(2\sqrt{\alpha}\right)^{n-2k}}{k!\,(n-2k)!}H_{n-2k}\left(x\sqrt{\alpha}\right)=(4\alpha-1)^{\frac{n}{2}}H_n\left(\frac{2\alpha x}{\sqrt{4\alpha-1}}\right),$$

where $\left[\frac{n}{2}\right]=\frac{n}{2}$ *or* $\frac{n-1}{2}$, *respectively, for even or odd* n *and* $\alpha>0$.

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Necessary conditions for retarded and free right ended variational problems

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We consider the following variational problem:

$$J(x(\cdot)) = \int_{t_0}^{t_1} L(t, x(t), x(t-h), \dot{x}(t), \dot{x}(t-h))dt + F(x(t_1)) \to \min_{x(\cdot)}, \tag{1}$$

$$x(t) = \varphi(t), t \in [t_0 - h, t_0]. \tag{2}$$

where R^n is an n-dimensional Euclidean space, $h = const > 0$, $t_1 - t_0 > h$, $x(t) \in KC^1([t_0 - h, t_1], R^n)$, $\varphi(t) \in C^1([t_0 - h, t_0], R^n)$ and $f(t, x, y, \dot{x}, \dot{y}) : [t_0, t_1] \times R^n \times R^n \times R^n \times R^n \to R := (-\infty, +\infty)$ is a continuous function, also $F(x) : R^n \to R$ is continuously differentiable function .Every function $x(\cdot)$ which satifies the condition (2) and belongs to the space $KC^1([t_0 - h, t_1], R^n)$ is called a admissible function. Let the function $\bar{x}(\cdot)$ be some admissible function in the problem (1)-(2) and $I^* \subseteq [t_0, t_1]$ be the set of points where the function $\dot{\bar{x}}(\cdot)$ is continuous.

In this paper the following theorem is proved.

Theorem 1. *Let the function $L(t, x, y, \dot{x}, \dot{y})$ be continuous on the set $I \times R^n \times R^n \times R^n \times R^n$. Then*

i) if the admissible function $\bar{x}(\cdot)$ is a strong local minimum in the problem (1)-(2), then the inequalities

$$Q(t, \lambda, \xi; \bar{x}(\cdot)) \geq 0, \forall (t, \lambda, \xi) \in I^* \times [0, 1) \times R^n,$$

$$\left(L(t, \bar{x}(t), \bar{x}(t-h), \dot{\bar{x}}(t) + \xi, \dot{\bar{x}}(t-h)) - \bar{L}(t)\right)\Big|_{t=t_1-0} + F_x^T(\bar{x}(t_1))\xi \geq 0, \forall \xi \in R^n$$

hold;

ii) if the admissible function $\bar{x}(\cdot)$ is a weak local minimum in the problem (1)-(2),then there exists a number $\delta > 0$ such that the inequalities

$$Q(t, \lambda, \xi; \bar{x}(\cdot)) \geq 0, \forall (t, \lambda, \xi) \in I^* \times \left[0, \frac{1}{2}\right) \times B_\delta(0),$$

$$\left(L(t, \bar{x}(t), \bar{x}(t-h), \dot{\bar{x}}(t) + \xi, \dot{\bar{x}}(t-h)) - \bar{L}(t)\right)\Big|_{t=t_1-0} + F_x^T(\bar{x}(t_1))\xi \geq 0, \forall \xi \in B_\delta(0).$$

hold.

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Transformation operators for the Schrödinger equation with an unbounded potential

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Let us consider the one-dimensional Schrödinger equation

$$-y'' + \theta(x)\, x^2 y + q(x)\, y = \lambda y,\ -\infty < x < \infty,\quad \lambda \in C, \tag{1}$$

where $\theta(x)$ is the Heaviside function and $q(x)$ is the real potential which satisfies the condition $\int\limits_{-\infty}^{+\infty} (1+|x|)\,|q(x)|\,dx < \infty$.

The following Theorem is proven.

Theorem. For any λ from the complex plane, the equation (1) has solutions $f_{\pm}(x,\lambda)$ representable as

$$f_{\pm}(x,\lambda) = \psi_{\pm}(x,\lambda) \pm \int\limits_{x}^{\pm\infty} K^{\pm}(x,t)\, \psi_{\pm}(t,\lambda)\, dt,$$

where the kernels $K^{\pm}(x,t)$ are continuous functions and satisfy the following relations:

$$K^{\pm}(x,t) = O\left(\sigma_0^{\pm}\left(\frac{x+t}{2}\right)\right), x+t \to \pm\infty,\ \sigma_0^{\pm}(x) = \pm\int\limits_{x}^{\pm\infty} |q(t)|\, dt,$$

$$K^{\pm}(x,x) = \pm\frac{1}{2}\int\limits_{x}^{\pm\infty} q\ (t)\ dt.$$

On the Iost representation of the Schrodinger equation with delta-shaped potential

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We consider a Schrodinger equation of the form

$$-y'' + \alpha\delta(x-a)\,y + q(x)\,y = \lambda^2 y,\ 0 < x < \infty,\ a > 0, \tag{1}$$

where α is a real function and $\delta(x)$ is Daric's delta function $q(x)$ is a real number, and satisfies the condition

$$\int_0^{+\infty} x\,|q(x)|\,dx < +\infty. \tag{2}$$

It should be noted that the equation (1) is equivalent (see[1,2]) to the following problem with a discontinuity condition

$$-y'' + \alpha\delta(x-a)\,y + q(x)\,y = \lambda^2 y,\ 0 < x < \infty,\ a > 0,$$

$$y'(a+0) - y'(a-0) = \alpha y(a).$$

It is easy to verify that for $q(x) = 0$ the equation has a solution in the form:

$$e_0(x,\lambda) = \begin{cases} e^{i\lambda x},\ x > a, \\ \left(1 + \frac{i\alpha}{2\lambda}\right) e^{i\lambda x} - \frac{i\alpha}{2\lambda} e^{i\lambda(2a-x)},\ 0 < x < a \end{cases}$$

The solution $e(x,\lambda)$ of the equation (1) is called Iost solution if it satisfies the condition $\lim\limits_{x\to+\infty} e(x,\lambda)\,e^{-i\lambda x} = 1$. Let $\sigma(x) = \int_x^\infty |q(t)|\,dt$, $\sigma_1(x) = \int_x^\infty \sigma(t)\,dt$.

Theorem. Let $q(x)$ satisfy the condition (2). For all λ from the closed upper halfplane the equation (1) has the Iost solution representable in the form

$$e(x,\lambda) = e_0(x,\lambda) + \int_x^{+\infty} K(x,t) e_0(t,\lambda) dt,$$

where the kernel $K(x,t)$ satissfy the relations

$$|K(x,t)| \le \frac{1}{2}\sigma\left(\frac{x+t}{2}\right) e^{\sigma_1(x)},$$

$$K(x,x) = \frac{1}{2}\int_x^\infty q(t)\,dt,$$

$$K'_x(a+0,t) - K'_x(a-0,t) = \alpha K(a,t)$$

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Markov-Nikol'skii type inequalities in regions with cusps

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Let $G \subset \mathbb{C}$ be a bounded Jordan region, $L := \partial G$; $P_n(z)$ - arbitrary algebraic polynomial, $\deg P_n \leq n$, $n \in \mathbb{N}$, and $h(z)$ be a generalized Jacobi weight function, defined as follows:

$$h(z) := \prod_{j=1}^{m} |z - z_j|^{\gamma_j}, \ \gamma_j > -2, \ j = 1, 2, ..., m, \ z \in G.$$

Let $0 < p \leq \infty$. For the Jordan region G, we introduce:

$$\begin{aligned} \|P_n\|_p \quad &: \quad = \|P_n\|_{A_p(h,G)} := \left(\iint_G h(z) |P_n(z)|^p \, d\sigma_z \right)^{1/p}, 0 < p < \infty, \\ \|P_n\|_\infty \quad &: \quad = \|P_n\|_{A_\infty(1,G)} := \max_{z \in \overline{G}} |P_n(z)|, \ p = \infty; \\ A_p(1,G) \quad &\equiv \quad A_p(G), \end{aligned}$$

where σ be the two-dimensional Lebesgue measure.

In this work, we study the following Markov-Nikol'skii type inequality for $p > 0$ and $m = 0, 1, 2, ..$ as the following:

$$\left\| P_n^{(m)} \right\|_\infty \leq c_1 \lambda_n(G, h, p) \|P_n\|_p,$$

where $c_1 := c_1(G, h, p) > 0$ is a constant independent of n and P_n, and $\lambda_n(G, h, p) \to \infty$, $n \to \infty$, depending on the geometrical properties of the region G, weight function h and p.

Fractal modeling of consolidation under filtration in water saturated soils

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The theory of consolidation of water-saturated soils, reflecting the process of convergence of ground particles and the reduction of pore volume, accompanied by the water displacement, attracts growing attention of researchers due to the increasing volume of construction ground and underground work in complex grounds. As a problem of unsteady filtration in a deformable medium, taking into account the variable porosity and ground permeability, its multiphase nature and the complex rheology of the phase components, it is sometimes not amenable to an analytical solution, and in this case, appropriate numerical methods are used to analyze the situation.

Recent studies allow us to view the fractal nature [1, 2, 4] of the structure of the ground mass. Currently, there are few such solutions, and they are obtained only for the simplest types of deformation, such as for a cylindrical or spherically symmetric type.

The construction of the theory of consolidation of the ground mass of the fractal structure is urgently required. However, it is natural that the resulting generalized system of equations will be many times more mathematically complex both in terms of its solution and analysis. Therefore, a step-by-step way of complicating the existing theories of ground consolidation, taking into account the fractal geometry of its structure, is preferable.

In the present paper, a two-dimensional equation for the consolidation of a two-phase, fully water-saturated ground (in the absence of a gas phase) is obtained, when the fractality of the pore space geometry is taken into account only in the Darcy law. In its output, we will adhere to the notation adopted in [4, 5].

The filtration law in the ground with fractal geometry is assumed according to [3] in the form:

$$u_x - \frac{n}{m}\upsilon_x = -k_x\left(\frac{\partial}{\partial x}\left(D^{\alpha}_{a+}H\right)(x) - i_0\right),$$

$$u_z - \frac{n}{m}\upsilon_z = -k_z\left(\frac{\partial}{\partial z}\left(D^{\beta}_{0+}H\right)(z) - i_0\right),$$

where,

$$\left(D^{\alpha}_{a+}H\right)(x) = \frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial x}\int_a^x \frac{H(\xi)\,d\xi}{(x-\xi)^a},$$

$$\left(D^{\beta}_{0+}H\right)(z) = \frac{1}{\Gamma(1-\beta)}\frac{\partial}{\partial z}\int_a^z \frac{H(\xi)\,d\xi}{(z-\xi)^a}$$

$$\frac{\partial\theta}{\partial t} = \frac{(1+e)}{\frac{de}{d\theta}}\left(\frac{\partial^2}{\partial x^2}\left(D^{\alpha}_{a+}H\right)(x) + \frac{\partial^2}{\partial z^2}\left(D^{\beta}_{0+}H\right)(z)\right)$$

This equation is called the basic equation of two-dimensional consolidation under the fractal filtration law, because it allows to take into account any computational model of the stress-strain state of the ground, i.e., any dependence of $\frac{de}{d\theta}$. The following equation for the two-dimensional consolidation of a fully water-saturated soil mass under the fractal filtration law was obtained.

$$\frac{\partial H}{\partial t} = C_v \left(\frac{\partial^2}{\partial x^2} \left(D^{\alpha}_{a+} H \right)(x) + \frac{\partial^2}{\partial z^2} \left(D^{\beta}_{0+} H \right)(z) \right), C_v = \frac{(1+e)(1+\xi)k}{2\gamma a}$$

We will evaluate the influence of the fractality of the filtration law on the consolidation process by using the example of solving the one-dimensional problem of consolidating the ground layer of fully water-saturated ground.

The consolidation equation for the one-dimensional case will have the form:

$$\frac{\partial H}{\partial t} = \frac{C_v}{\Gamma(1-\beta)} \frac{\partial^3}{\partial z^3} \int_0^z \frac{H(\xi, t)}{(z-\xi)^{\beta}} d\xi$$

To solve the problem, we use the numerical method of finite differences.

The two-dimensional consolidation equation for two-phase, fully water-saturated soil (in the absence of a gas phase), when the fractality of the pore space geometry is taken into account only in the Darcy filtration law, was derived from the research results.

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Flow of heterophase mixtures in gas pipes and methods to control gas-dynamic parameters

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Non-equilibrium processes and phase transitions determine the optimal thermal and hydro and gas dynamics modes of flow of the gas-liquid mixture in pipelines under development of gas and gas condensate fields. The piston mode of flow in pipelines of complex geometric configuration can form due to gravitational segregation of the liquid in the pipeline, which, in turn, blocks the free flow of gas and leads to a gradual increase in pressure. Predicting the possibility of piston flow formation, oscillation period and peak flow values is important from a practical point of view to select the proper conditions for its operation.

Both identification models and thermodynamic approaches are used to estimate these parameters. The use of both methods shows that, as the simplest model for describing nonequilibrium phenomena in a gas-liquid mixture, a thermodynamic relation of the type

$$\frac{1}{C_0^2}\left(p-p_0+\theta_1\frac{\partial p}{\partial t}\right)=\rho-\rho_0+\frac{C_\infty^2}{C_0^2}\theta_2\frac{\partial p}{\partial t}, \tag{1}$$

where $p,\ \rho$ – the current values of the pressure and density of the gas-liquid mixture; p_0,ρ_0- the initial values of the pressure and density of the gas-liquid mixture; C_0,C_∞- the propagation velocity of the wave of disturbances in slow and fast-flowing processes; θ_1,θ_2-relaxation times for the pressure and density of the gas-liquid mixture.

Dynamic inhomogeneities of the turbulent fluid in them induce flowing (ejection) to the jet, which leads to unsteady non-equilibrium flow. Formation in boundary layer of liquid and intense underflow by ordinary turbulent jets and jets with twisting also leads to strongly pulsating flow [1]. The wave front interacting with boundary layer undergoes changes and so-called lambda-like jumps with impulses are formed [2, 3].

The pressure change at such pulsation modes can be represented similarly to [4, 5] and described by the law $e^{-\sin\left(\frac{t}{Q_1}\right)}$.

The paper uses the following form of the equation for the momentum of the mixture

$$\rho_m\frac{\partial v_m}{\partial t}+2\rho_m v_m\frac{\partial v_m}{\partial z}=\rho_m g\cos\theta-\frac{2f\rho_m v_m\left|v_m\right|}{d}, \tag{2}$$

where $\rho_m=\omega_i\rho_i+\omega_g\rho_g$ is the density of the mixture, $f=f\left(\omega_g,v_m,p\right)$ – is the coefficient of friction, $d$is the diameter of the pipe, $\theta$is the angle between the pipe axis and the vertical. $\varepsilon_1,\varepsilon_2$coefficients taking the values 0 and 1 are introduced to consider various variants of the model.

The paper proposes the methods and process control of the permissible values of wave and vibration loads for various types of transported media.

Analyzing the methods and designs of foam generation devices, a method of foam generation was developed to give elastic properties to liquids [6]. In order to prevent pulsations and water shocks, as well as to clean pipelines from accumulations, a special foaming agent (solid surfactant) is proposed, installed to generate foam in special branches from the main gas pipeline - looping.

The considered approach makes it possible to analyze the dynamics of the pipeline with an arbitrarily complex and rapid change in internal pressure.

The case of multi-component systems flows in pipes under conditions of pulsating the pressure and density of a gas-liquid mixture is studied using elements of the drift model for the problem of the bubble flow regime in a long pipe.

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Analysis of the study of some topics in the course of algebra

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General education schools provide a system of education in basic school subjects, including mathematics, and are not fully satisfied. Despite the fact that a lot of time has been allocated in the curriculum to study the main subjects of the school, students ' knowledge is still formally and quickly forgotten. Analysis of scientific and methodological literature, school practice and results of pedagogical research one of the main contradictions of modern education is the contradiction between the needs of a changing society and the traditions of teaching school subjects. For a long time, the improvement of the educational process was carried out only by changing the content of educational material, and there are large reserves in the development of new forms and methods of teaching.When teaching elements of mathematical analysis in school, special attention is paid to the concepts of "derivative" and"initial function". This is due to the widespread use of these concepts in both school mathematics and physics. In mathematics, it is used to study the derivative of a function, and the first function is used when calculating the area of curved shapes.First, we will present all the new concepts introduced to the subject of algebra and the beginning of analysis in high schools . In the Great Soviet encyclopedia, it is written: "mathematical analysis is a science that works with the study of functions; differential and integral calculus is a science that studies intermediate and differential functions, as well as the science that solves differential and integral equations". On the one hand, the concept of a function is "basic", on the other hand, there are requirements for differential and integral calculations. Hence, the basic concepts of elements of mathematical analysis include concepts related to differential and integral calculations, namely:

1 the limit circuit; 2 complete the circuit; 3 derivative; 4. definite integral; 5. limit of the function at a point; 6.continuous function; 7. antiderivative; 8. the properties of the limit;	9.chain restriction; 10. limit chain and limit of a function; 11. the differentiation formula; 12. increasing the argument and function; 13. rules of differentiation; 14.equation of touch to the graph of functions;15.functions extramammary; 16.investigation of the function for monotony.

Categories for comparison	A. Abylkasymova "Algebra and the beginning of analysis".	Shynybekov A. "Algebra and the beginning of analysis"
Place of introduction of the derived concept (in class 10)	The product is entered in the second half of class 10.	The derivative is introduced in the first half of class 10, so the derivative generalizes and systematizes the properties of various functions - trigonometric, logarithmic, power, and others ...
Mathematical concepts used to familiarize yourself with a derived concept	To familiarize yourself with the derived concept, the concepts of sequence threshold, geometric progression, and function limits are included. These concepts are	The study of the topic "derivative" begins with the introduction of the concept of multiplication of functions and the rules for calculating it from the concept. Then differentiable functions are considered. At a
	carefully analyzed and examples of developing problem-solving skills are provided.	point, a function is defined that is differentiated using limit values. Differentiation of functions is proved by an example.
Methodological features of the implementation of the definition of " derivative»	Two different physical and geometric problems are considered, the process of solving which leads to the appearance of a new mathematical model.	First, we consider problems with solutions for increasing the function based on which a derived definition is introduced.
Introduction of the geometric value of a derived function	This topic begins with a geometric meaning. Set schedules and detailed characteristics. It is formulated in the form of problems associated with the solution.	The whole concept of illustrative examples issued by the
Methodological features of the study of the use of derivatives in the study of functions	The tutorial describes the algorithm for investigating the function: 1. Find the product of the function 2. Find constant and critical points; 3. Determine the derived features from the obtained intervals; 4. Create based on theorems	The textbook has many theorems on this topic. A clear algorithm is not provided.

We will conduct a comparative analysis of textbooks of secondary schools- this is Abylkasymova A. "Algebra and the beginning of analysis" (grade 10) and Shynybekov A. " Algebra and the beginning of analysis (10 grade). The results of the analysis are shown in the table.

In our opinion, the theoretical material on the introduction of mathematical analysis concepts in the textbook of A. E. Abylkasymov is presented at an accessible level, students can independently familiarize themselves with the material, collect examples provided for textbooks, and solve their tasks. Tasks were presented at different levels and in large numbers, there was not enough time to solve them in full. On the other hand, the teacher has the opportunity to provide students with additional or the most successful in mathematics additional work in the classroom and homework. A. textbook of Shynybekov et al. for training at the basic and specialized levels, which allows you to successfully organize work with students of various levels of training. But there is a question: Is it possible to write an equally accessible language for students of basic and specialized levels? In General, the textbook is written in a language that is accessible to students, and contains many examples of the formulas and main tasks being studied. The tutorial contains additional material for repetition.

Limit theorems for the Markov Random walk described by an autoregressive process $AR(1)$ for nonlinear boundaries

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Suppose that $\{\xi_n,\ n \geq 1\}$ is a sequence of independent identically distributed random variables defined on some probability space (Ω, F, P). As is known [1,2], the first-order autoregressive process $AR(1)$ is determined using a recurrence relation of the form $X_n = \beta X_{n-1} + \xi_n, n \geq 1$, where $\beta \in R = (-\infty, +\infty)$ is some fixed number and it is assumed that the initial value of X_0 does not depend on innovation $\{\xi_n\}$.

Consider a family of first passage times

$$\tau_\alpha = inf\{n \geq 1 : T_n \geq f_\alpha(n)\} \tag{1}$$

of the Markov random $T_n = \sum_{k=1}^{n} X_k X_{k-1}, n \geq 1$ for the nonlinear boundary $f_a(t), a > 0, t > 0$. Assume that $\inf\{\emptyset\} = \infty$.

In this work, we prove the central limit theorem for the Markov random walk $T_n, n \geq 1$ and family (1).

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The propagation of non – stationary waves in a half space with a built-in cylinder

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Investigated non – stationary dynamics of wave propagation in mixed regions.

It is assumed that an elastic cylinder, made of another material is built into the elastic half space, and the end platform of the cylinder is subjected to the impact forces. Problem is solved with applying the integral Laplase and Fourier Transforms, with leads to images, with 4^{th} rank determinants.

To construct solutions in inverse transformations, the methods developed in works [1,2] were used, which made it possible to obtain analytical expressions for all required quantities. Graph of the dependence of the longitudinal velocity along the axis of the cylinder on the subsequent values of time are obtained. The results confirm all theoretical expectations.

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Riesz-Thorin type theorems in $S^{l}_{p,\theta,\varphi,\beta}B(G)$

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In the abstract, we study differential properties of functions from intersections of Besov-Morrey spaces with dominant mixed derivatives $S^{l^{\mu}}_{p_\mu,\theta_\mu,\varphi,\beta}B(G_\varphi)$ $(\mu = 1,2,\ldots,M)$ with finite norm $(m_j > l_j > k_j \geq 0, j = \{1,2,...,n\} = e_n)$

$$\|f\|_{S^{l}_{p,\theta,\varphi,\beta}B(G_\varphi)} = \sum_{e\subseteq e_n} \|D^{l^e} f\|_{L_{p,\theta,\varphi,\beta}(G_\varphi)},$$

where

$$\|f\|_{L_{p,\theta,\varphi,\beta}(G_\varphi)} = \left\{ \int\limits_{0^e}^{t_0^e} \left[\frac{\triangle^{m^e}(\varphi(t), G_{\varphi(t)}) D^{k^e} \|f\|_{p,\varphi,\beta}}{\prod\limits_{j\in e} (\varphi_j(t))^{l_j-k_j}} \right]^{\theta} \prod_{j\in e} \frac{d\varphi_j(t)}{\varphi_j(t)} \right\}^{\frac{1}{\theta}}.$$

$$\|f\|_{p,\varphi,\beta;G} = \|f\|_{L_{p,\varphi,\beta}(G)} = \sup_{\substack{x\in G,\\ t_j>0,\\ j\in e_n}} \left(\prod_{j\in e_n} (\varphi_j([t_j]_1))^{-\beta_j} \|f\|_{p,G_{\varphi(t)}(x)} \right),$$

$l \in (0,\infty)^n$, $l^e = (l_1^e, l_2^e, \ldots, l_n^e)$, $l_j^e = l_j$ $(j \in e)$; $l_j^e = 0$, $(j \in e_n - e)$; $m \in N^n$; $k \in N_0^n$; $1 \leq p_\mu < \infty$, $1 \leq \theta_\mu \leq \infty$ $(\mu = 1,2,...,N)$; $\beta \in [0,1]^n$; $\varphi(t) = (\varphi_1(t_1), \varphi_2(t_2), \ldots, \varphi_n(t_n))$, $\varphi_j(t_j) > 0$ is continuously differentiable functions $\varphi_j'(t_j) > 0$ $(j \in e_n)$; $\lim\limits_{t_j\to+0} \varphi_j(t_j) = 0$, $\lim\limits_{t_j\to+\infty} \varphi_j(t_j) = K_j$, $0 < K_j \leq \infty$ $(j \in e_n)$; $t_0 = (t_{01}, \ldots, t_{0n})$ is fixed positive vector.

By the method of integral representations, we proved the following statements:

1) $D^\nu : \bigcap\limits_{\mu=1}^{M} S^{l_\mu}_{p_\mu,\theta_\mu,\varphi,\beta}B(G_\varphi) \to L_{q,\psi,\beta_1}(G)(C(G))$ is holds; 2) It is also proved that for the functions from space $\bigcap\limits_{\mu=1}^{M} S^{l_\mu}_{p_\mu,\theta_\mu,\varphi,\beta}B(G_\varphi)$ the generalized derivatives $D^\nu f$ satisfy the Hölder condition in the metric $L_q(G)$ and $C(G)$.

Approximation of functions by singular integrals

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Let R^n be n-dimensional Euclidean space of points $x = (x_1, x_2, ..., x_n)$, $B(a,r) :=$ $\{x \in R^n \colon |x-a| \le r\}$ –be a closed ball in R^n of radius $r > 0$ centered at the point $a \in R^n$, N be the set of all natural numbers, $\nu = (\nu_1, \nu_2, ..., \nu_n)$, $x^\nu = x_1^{\nu_1} \cdot x_2^{\nu_2} \cdots x_n^{\nu_n}$, $|\nu| = \nu_1 + \nu_2 + ... + \nu_n$, where $\nu_1, \nu_2, ..., \nu_n$ are non-negative integers. Denote the class of all locally p-power summable functions defined on R^n by $L^p_{loc}(R^n)$ $(1 \le p < \infty)$, the class of all locally bounded functions defined on R^n by $L^\infty_{loc}(R^n)$.

Let $f \in L^1_{loc}(R^n)$, $k \in \mathrm{N} \bigcup \{0\}$. Let us consider the polynomial

$$P_{k,\mathrm{B}(a.r)} f(x) := \sum_{|\nu| \le k} \left(\frac{1}{|\mathrm{B}(a,r)|} \int_{\mathrm{B}(a,r)} f(t)\, \varphi_\nu \left(\frac{t-a}{r} \right) dt \right) \varphi_\nu \left(\frac{x-a}{r} \right),$$

where $|B(a,r)|$ denotes the volume of the ball $B(a,r)$ and $\{\varphi_\nu\}$, $|\nu| \le k$, is an orthonormed system obtained from applications of orthogonalization process with respect to the scalar product

$$(f,g) := \frac{1}{|B(0,1)|} \int_{B(0,1)} f(t) g(t) dt$$

to the system of power functions $\{x^\nu\}$, $|\nu| \le k$, located in partially lexicographic order.

Let

$$f \in L^p_{loc}(R^n), 1 \le p \le \infty, 1 \le q \le \infty, m_f^k(x;r)_p :=$$

$$= \sup \left\{ \Omega_k (f, \mathrm{B}(x,t))_p \ : 0 < \ t \le r \right\} (x \in R^n, r > 0), m_f^k(r)_{pq} :=$$

$$\begin{cases} \left\| m_f^k(\cdot;r)_p \right\|_{L^q(R^n)} & if\ 1 \le q < \infty, \\ \sup \left\{ m_f^k(x;r)_p : x \in R^n \right\} & if q = \infty. \end{cases}$$

where $\Omega_k (f, \mathrm{B}(x,r))_p := |B(x,r)|^{-1/p} \left\| f - P_{k-1,B(x,r)} f \right\|_{L^p(B(x,r))}$ $(x \in R^n,\ r > 0)$.

Introduce the singular integral

$$S_{k,r}(f;K)(x) = \int_{R^n} K_r(x-t) \left[f(t) - P_{k-1,B(x,r)} f(t) \right] dt + P_{k-1,B(x,r)} f(x),$$

where $K \in L^1(R^n)$, $K_r(x) := r^{-n} K\left(\frac{x}{r}\right)$, $r > 0$, $k \in N$, $x \in R^n$. Let $\psi(x) :=\ = esssup\{|K(y)| : |y| \ge |x|\}$, $\varphi(|x|) := \psi(x)$, $k \in N$. By Λ_k we will denote the class of all functions $K(x)$ measurable in R^n such that $\psi \in L^1(B(0,1))$, $|x|^{k-1} \cdot \psi \in L^1(R^n \backslash B(0,1))$. It is easy to see that $\Lambda_k \subset L^1(R^n)$.

Theorem. Let $K \in \Lambda_k$, $k \in N$, $f \in L^p_{loc}(R^n)$, $1 \le p,q \le \infty$. Then under the convergence of the integrals in the right hand side, almost everywhere in R^n there exists finite limit $s_{k,f}(x) = \lim\limits_{r\to 0} P_{k-1,B(x,r)} f(x)$ and the following inequality is valid

$$\|S_{k,r}(f;K) - s_{k,f}\|_{L^q(R^n)} \le$$

$$\le c(n,\psi,k)\left(m_f^k(r)_{pq} + \int_0^r \frac{m_f^k(t)_{pq}}{t} dt + \int_0^\infty x^{n-1}\varphi(x)\, m_f^k(4rx)_{pq}\, dx + \right.$$

$$+ \int_0^r \frac{m_f^k(t)_{pq}}{t}\left(\int_0^{t/r} x^{n-1}\varphi(x)\,dx\right) dt +$$

$$\left. + r^{k-1}\int_r^\infty \frac{m_f^k(t)_{pq}}{t^k}\left(\int_{t/r}^\infty x^{n+k-2}\varphi(x)\,dx\right) dt \right),$$

where $c(n,\psi,k)$ is a positive constant dependent only on n, ψ and k.

Asymptotics of the solution to a boundary value problem for a singularly perturbed differential equation of arbitrary order

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We study the following boundary value problem

$$(-1)^m \varepsilon^{2m} \frac{d^{2m+1}u}{dt^{2m+1}} + \frac{du}{dt} - au = f(t), \tag{1}$$

$$u|_{t=0} = \left.\frac{du}{dt}\right|_{t=0} = \cdots = \left.\frac{d^m u}{dt^m}\right|_{t=0} = 0,$$

$$\left.\frac{d^{m+1}u}{dt^{m+1}}\right|_{t=1} = \left.\frac{d^{m+2}u}{dt^{m+2}}\right|_{t=1} = \cdots = \left.\frac{d^{2m}u}{dt^{2m}}\right|_{t=1} = 0, \tag{2}$$

where $\varepsilon > 0$ ia s small parameter, m is an arbitrary natural number, $a > 0$ is a constant, $f(t)$ is a given smooth function.

The following theorem was proved.

Theorem. Let the function $f(t)$ have continuous derivatives to the $(2m + 2n + 2)$ -th order inclusively, on the interval [0,1]. Then the following asymptotic representation is valid for solving problem (1),(2)

$$u = \sum_{i=0}^{n} \varepsilon^i W_i + \sum_{s=0}^{n+m-1} \varepsilon^{1+s} V_s + \sum_{s=0}^{n+m-1} \varepsilon^{1+m+s} \psi_s + \varepsilon^{n+1} z,$$

where the function w_i is determined by the first iterative process, v_s, ψ_s are boundary layer type functions near the boundaries $t = 0$ and $t = 1$ that are determined by appropriate iterative processes, $\varepsilon^{n+1} z$ is a residual term, and for z the following estimation is valid:

$$\varepsilon^m \left(\left.\frac{d^m z}{dt^m}\right|_{t=1} \right)^2 + \left(z|_{t=1} \right)^2 + c_1 \int_0^1 z^2 dt \le c_2 \varepsilon^{2(n+1)},$$

where $c_1 > 0, c_2 > 0$ are constants independent of ε.

Asymptotics of the solution to a boundary value problem for a singularly perturbed quasilinear differential equation of second order

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In the paper we consider the following boundary value problem

$$-\varepsilon^p \frac{d}{dt}\left(\frac{du}{dt}\right)^p - \varepsilon \frac{d^2u}{dt^2} + \frac{du}{dt} + au = f(t), \tag{1}$$

$$u|_{t=0} = 0 \; u|_{t=1} = 0, \tag{2}$$

where $\varepsilon > 0$ is a small parameter, $p = 2k + 1, k$ is an arbitrary natural number, $a > 0$ is a constant, $f(t)$ is a given smooth function.

The goal of the paper is to construct complete asymptotics of the solution of boundary value problem (1),(2) with respect to the small parameter. To do this, we carry out iterative processes and prove the following theorem.

Theorem. Let the function $f(t)$ have continuous derivatives to the $(2m + 2n + 2)$-th order inclusively, on the interval [0,1]. Then for solving problem (1),(2), we have the asymptotic representation

$$u = \sum_{i=0}^{n} \varepsilon^i W_i + \sum_{j=0}^{n+1} \varepsilon^j V_j + z,$$

where the functions W_i are determined by the first iterative process, the functions v_j are boundary layer type functions near the boundary $t = 1$ and are determined by the second iterative process, z is a residual term, and the estimation

$$\varepsilon^{2k+1} \int_0^1 \left(\frac{\partial z}{\partial t}\right)^{2k+1} dt + \varepsilon \int_0^1 \left(\frac{\partial z}{\partial t}\right)^2 dt + c_1 \int_0^1 z^2 dt \le c_2 \varepsilon^{2(n+1)}$$

is valid for it, here $c_1 > 0, c_2 > 0$ are constants independents of ε.

On Riemann problem in weighted Smirnov classes

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Weighted Smirnov classes with power weight in bounded and unbounded domains are defined in this work. Nonhomogeneous Riemann problem with a measurable coefficient whose argument is a piecewise continuous function is considered in these classes.

By $L_{p,\omega}(\Gamma)$ we denote the weighted Lebesgue space of functions endowed with the norm $\|\cdot\|_{p,\omega}$:

$$\|f\|_{L_{p,\omega}(\Gamma)} = \left(\int |f(\xi)|^p \omega(\xi) |d\xi|\right)^{\frac{1}{p}}.$$

Let $G(\xi) = |G(\xi)| e^{i\theta(\xi)}$ be complex-valued functions on the curve Γ. We make the following basic assumptions on the coefficient $G(\cdot)$ of the considered boundary value problem and Γ:

(i) $|G(\cdot)|^{\pm 1} \in L_\infty(\Gamma)$;

(ii) $\theta(\cdot)$ *is piecewise continuous on* Γ, *and* $\left\{\xi_k, \, k = \overline{1,r}\right\} \subset \Gamma$ *are discontinuity points of the function* $\theta(\cdot)$.

We impose the following condition on the curve Γ.

(iii) Γ *is either Lyapunov or Radon curve (i.e. it is a limited rotation curve) with no cusps. Direction along* Γ *will be considered as positive, i.e. when moving along this direction the domain* D *stays on the left side. Let* $a \in \Gamma$ *be an initial (and also a final) point of the curve* Γ. *We will assume that* $\xi \in \Gamma$ *follows the point* $\tau \in \Gamma$, *i.e.* $\tau \prec \xi$, *if* ξ *follows* τ *when moving along a positive direction on* $\Gamma \backslash a$, *where* $a \in \Gamma$ *represents two stuck points* $a^+ = a^-$, *with* a^+ *a beginning, and* a^- *an end of the curve* Γ.

Let $D^+ \subset \mathbb{C}$ be a bounded domain with the boundary $\Gamma = \partial D^+$, which satisfies the condition (iii). Denote by $E_p(D^+)$, $1 < p < \infty$, a Smirnov Banach space of analytic functions in D^+ with the norm $\|\cdot\|_{E_p(D^+)}$:

$$\|f\|_{E_p(D^+)} =: \left\|f^+\right\|_{L_p(\Gamma)}, \forall f \in E_p\left(D^+\right), \tag{1}$$

where $f^+ = f/_\Gamma$ are non-tangential boundary values of the function f on Γ.

Similarly we define the Smirnov classes $E_p(D^-)$ in unbounded domain D^- with boundary $\Gamma = \partial D^-$ and with the norm

$$\|f\|_{E_p(D^-)} =: \left\|f^-\right\|_{L_p(\Gamma)}, \forall f \in E_p\left(D^-\right),$$

where $f^- = f/_{\Gamma^-}$ are non-tangential boundary values of the function f on Γ.

Based on the norm (1), we define the weighted Smirnov class. Let $\rho \in L_1(\Gamma)$ be some weight function. Define weighted Smirnov class $E_{p,\rho}(D^+)$:

$$E_{p,\rho}\left(D^+\right) \equiv \left\{ f \in E_1\left(D^+\right) : \left\|f^+\right\|_{L_{p,\rho}(\Gamma)} < +\infty \right\},$$

and let

$$\|f\|_{E_{p,\rho}(D^+)} = \left\|f^+\right\|_{L_{p,\rho}(\Gamma)}. \tag{2}$$

Similarly we define the Smirnov classes in unbounded domain. Let $D^- \subset \mathbb{C}$ be an unbounded domain containing infinitely remote point (∞). Denote by ${}_mE_1(D^-)$ a class of functions from $E_1(D^-)$ which are analytic in D^- and have an order $k \leq m$ at infinity, i.e. the function $f \in E_1(D^-)$ has a Laurent decomposition $f(z) = \sum_{k=-\infty}^{m} a_k z^k$ in the vicinity of the infinitely remote point $z = \infty$, where m is some integer.

Consider the nonhomogeneous Riemann problem

$$F^+(z(s)) - G(z(s))\,F^-(z(s)) = g(z(s))\ ,\ s \in (0,S)\,, \tag{3}$$

where $g \in L_{p,\rho}(\Gamma)$ is a given function. By the solution of the problem (3) we mean a pair of functions $(F^+(z)\,; F^-(z)) \in E_{p,\rho}(D^+) \times_m E_{p,\rho}(D^-)$, whose boundary values $F^{\pm}$ on Γ a.e. satisfy (3). Let us introduce the following weight function

$$\nu(s) =: \sigma^p(s)\,\rho(z(s))\ ,\ s \in (0,S)\,, \tag{4}$$

where the weight function $\rho(\cdot)$ is defined by the expression

$$\rho(s) = \rho(z(s)) = \prod_{k=0}^{m_0} |z(s) - z(t_k)|^{\alpha_k}\,. \tag{5}$$

We will assume that the weight $\rho(\cdot)$ satisfies the condition

$$\alpha_i < \frac{q}{p}\,,\ \ i = \overline{0, m_0}. \tag{6}$$

Let's find a particular solution $F_1(z)$ of nonhomogeneous problem that corresponds to the argument $\theta(\cdot)$. Let $Z_\theta(z)$ be a canonical solution of homogeneous problem corresponding to the argument $\theta(\cdot)$. Consider the following piecewise analytic function

$$F_1(z) \equiv \frac{Z_\theta(z)}{2\pi i} \int_\Gamma \frac{g(\xi)\,d\xi}{Z_\theta^+(\xi)\,(\xi - z)},\ z \notin \Gamma. \tag{7}$$

It is proved the following

Statement. Let the coefficient $G(z(s))$, $0 \leq s \leq S$, of the problem (3) and the curve $\Gamma = z([0,S])$ satisfy the conditions (i)-(iii). Let the weight function $\rho(\cdot)$ defined by the expression (5) and the quantity $\{\beta_k\}_0^l$ are defined by

$$\beta_k = -\frac{p}{2\pi} \sum_{i=0}^{r} h_i \chi_{T_k}(s_i) + \sum_{i=0}^{m_0} \alpha_i \chi_{T_k}(t_i)\,, k = \overline{0, l}.$$

Let the conditions

$$-1 < \beta_k < \frac{p}{q}, k = \overline{0, l}.$$

and (6) hold. Then the analytic function $F_1(\cdot)$ defined by the Cauchy integral (7) is a particular solution of nonhomogeneous Riemann problem (3) in weighted Smirnov classes $E_{p;\rho}(D^+) \times_{-1} E_{p;\rho}(D^-), 1 < p < +\infty$.

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On new sequence space defined by a modulus and infinite matrix

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In [4] Ruckle present the following notion of modulus function:
Definition 1. *A function $f : [0, \infty) \to [0, \infty)$ is called a modulus function provided that*

1. $f(x) = 0$ *if and only if* $x = 0$,
2. $f(x + y) \leq f(x) + f(y)$ *for all* $x \geq 0$ *and* $y \geq 0$,
3. f *is increasing, and*
4. f *is continuous from the right of* 0.

Observe by (2) and (4) it follows instantly that f is continuous on $[0, \infty)$. Moreover, by (2) for all natural number n, $f(nx) \leq nf(x)$. Consider next the following type of matrix transformation.
Definition 2. *Let $A = [a_{n,k}]$ denote a summability transformation [3] that maps complex sequences x into the sequence Ax where the n-th term of Ax is as follows:*

$$[Ax_n] = \sum_{k=1}^{\infty} a_{n,k} x_k$$

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Approximation of classes of harmonic bounded functions by incomplete Fejer means

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Let hB^1 be a class of harmonic in disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ functions, such that $\|f'\|_\infty < 1$. The Fejer means $\sigma_n[f]$ for function f are defined by

$$\sigma_n[f](z) = \sum_{|k|\le n-1} \left(1 - \frac{|k|}{n}\right) \widehat{f}_k \epsilon_k(z), \quad \widehat{f}_k = \frac{f^{(k)}(0)}{k!}, \quad \epsilon_k(z) = \begin{cases} \overline{z}^{|k|}, & k \le -1, \\ z^{|k|}, & k > 0. \end{cases}$$

Every function $f \in C_q := hB^1|_{|z|=q}$ given by the Poisson integral $f(x) = \frac{1}{\pi}\int_{-\pi}^{\pi} \varphi(x + t) \sum_{k=1}^{\infty} q^k \cos kt\, dt$, where $\varphi \in W^1$. In our research we study the asymptotic behavior of upper bounds of deviations of incomplete Fejer means $\widetilde{\sigma}_n[f](x) = \frac{n}{n+1}\sigma_n[f](x)$ on classes C_q. Particularly, we obtain the following statement.

Theorem 1. *Let $q \in (0; q_0]$, $q_0 = (17/27 + \sqrt{11/27})^{1/3} + (17/27 - \sqrt{11/27})^{1/3} \approx 0,54$. The asymptotic equality is true as $n \to \infty$*

$$\max_{f \in C_q} \|f - \widetilde{\sigma}_n\|_C = \frac{4}{\pi(n+1)} \left(\frac{q}{1+q^2} + \mathrm{arctg} q\right) + O(1)\frac{q^n}{n},$$

where $O(1)$ — quantity, uniformly bounded by n.

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Determination of the structural-load and solids capacity under creep taking into account external physical fields and impacts

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The mathematical theory of creep, one of the creators of which is Yu.N. Rabotnov, allowed to generalize the disparate experimental results [1]. As a result, the theory of creep took the form of mechanical equations of state with state parameters defined by the corresponding system of kinetic equations. By introducing state parameters such as creep deformation, damage, scattered work, etc. Rabotnov identified the main possibilities of creep modeling. The kinetic equations were refined and generalized later, and the limits of the applicability of various variants of kinetic equations were established [2].

In this paper, the structural-load capacity of thin-walled structural elements and solids during creep under of external physical fields and impacts is investigated. The variational principle of the mixed type is applied, as in the [3], compiled for geometrically nonlinear problems

$$J = \int_v (\dot{\sigma}^{ij}\dot{\varepsilon}_{ij} + \tfrac{1}{2}\,\sigma^{ij}\,\dot{u}_{k,i}\,\dot{u}^k_{,j} - \tfrac{1}{2}\left(\dot{\varepsilon}^m_{ij} + 2\dot{p}_{ij} + \dot{\theta}\delta_{ij}\right)\dot{\sigma}_{ij} + \lambda_\omega\left(\tfrac{1}{2}\dot{\omega}^2 - \dot{\omega}\phi\right) +$$
$$\lambda_A\left[\tfrac{1}{2}\dot{c}^2 - \dot{c}div\,(D\nabla c) - kc\dot{c}\right])dV - \int_{\sum_\sigma}\vec{T}^i\dot{u}_i d\Sigma - \int_{\sum_u}\vec{T}^i\left(\dot{u}_i - \dot{\bar{u}}_i\right)d\Sigma$$

formulated for plastic bodies and composites for creep problems, taking into account the damage, diffusion process, and irradiation with neutron flux. Here the independent quantities are $\dot{\sigma}^{ij}$, $\dot{u}_i$, $\dot{\omega}$ and $\dot{c}$, and $\lambda_\omega = \lambda_\omega\left(\sigma^{\alpha\beta},\, \varepsilon_{\alpha\beta}\right)$ and $\lambda_A = \lambda_A\left(\sigma^{\alpha\beta},\, \varepsilon_{\alpha\beta}\right)$ are the weight functions whose values are chosen depending on the type of interpolation functions to refine the approximations.

The creep process is accompanied by instantaneous elastic and plastic deformations under high temperatures, and the creep relations are formulated taking into account several state parameters [4]. By virtue of the main lemma of the calculus of variations for the mathematical model of the problem we obtain the Euler equation of the following form

$$\begin{array}{l}
\frac{d}{dt}\left[\sigma_{ij}\left(\delta^k_j + u,^k_j\right)\right]_{,j} = 0 \\
\frac{d}{dt}\left[\sigma_{ij}\left(\delta^k_j + u,^k_j\right)\right]\nu_i = \dot{T}^k\ ,\ x^k \in \Sigma_\sigma\ ,\ \dot{u}_i = \dot{\bar{u}}_i\ ,\ x^k \in \Sigma_u \\
2\varepsilon_{ij} = u_{i,j} + u_{j,i} + u_{k,i}u^k_{,j} \\
\dot{\varepsilon}_{ij} = \left\{C_{ijkl}\sigma^{kl} + \theta\delta_{ij}\right\}^{\cdot} + \dot{p}_{ij}\left(\varepsilon_{\alpha\beta}, \sigma^{\alpha\beta}, \omega, c\right) \\
\dot{c} = div\,(D\nabla c) - kc \\
\dot{\omega} = \phi\left(\sigma^{\alpha\beta}, \omega, c\right)
\end{array}$$

In order to overcome the difficulty of solving the variational problem using direct methods, abandoning the exact satisfaction of the kinetic equations, we replace them with approximate integral relations

$$\int_V \lambda_\omega \left[\dot{\omega} - \phi \left(\sigma^{\alpha\,\beta},\, p_{i\,j},\, \omega\right)\right]^2 d\,V \approx 0$$

$$\int_V \lambda_c \left[\dot{c} - div \left(D\,\nabla\, c\right) + k\,c\right]^2 d\,V \approx 0$$

where the damageability functions and the concentration level of the corrosive medium are calculated in the form of a series

$$\omega = \sum_{k=1}^{p} a_k \; (t) \; \psi_k \; (x_j)\,; \;\; a_k \; (0) = 0; \;\; c = \sum_{k=1}^{m} c_k \; (t) \; \eta_k \; (x_j)\,; \, c_k \; (0) = 0$$

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Optimization of waterflooding performance in a hydrocarbon reservoir under an unstable oil displacement front

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Non-stationary flooding of oil-saturated reservoirs has a long-standing and durable place as the main secondary method of oil production and maintenance of reservoir pressure in the development of most oil reservoirs. The multidimensional geological and hydrodynamic models of multi-phase flow solve the main long- and medium-term problems of oil and gas-saturated reservoir management, including water flooding as a secondary method of increasing oil recovery. At the same time, important short-term operational flooding tasks cannot be solved using models, the main purpose of which is to calculate options for rational reservoir development for the purposes of drafting project documentation [1-2].

A complete qualitative theory of such dynamical systems is constructed, including an exhaustive analysis of all their singular points and features of solutions. The properties of solutions in the phase planes of the parameters and in time themselves are investigated. It is shown that the discriminant of polynomials are the control parameters controlling the essential properties of solutions, a classification of solutions is proposed depending on the values and signs of the discriminant and belonging to a specific family D^{++}, D^{+-}, D^{-+}or D^{--} [3-5].

In order to construct a qualitative theory of polynomial quadratic DS and to introduce a discriminant criterion of qualitative theory, we specify, following, the definitions of D^{++}, D^{-+}, D^{+-} and D^{---} classes as sets of $\{f_o, f_w\}$ pairs of polynomials $f_o\left(Q_O\right) = a_o {Q_O}^2 + b_o Q_O + c_0$ and $f_w\left(Q_w\right) = a_w {Q_w}^2 + b_w Q_w + c_w$ such that

$D^{++} : D_o = {b_o}^2 - 4a_o c_o > 0$ and $D_w = {b_w}^2 - 4a_w c_w > 0$; $D^{-+} : D_o < 0$ and $D_w > 0$; $D^{+-} : D_o > 0$ and $D_w < 0$; $D^{--} : D_o < 0$ and $D_w < 0$.

Table 1. General solutions of the equation $\frac{dQ_w}{dQ_O} = \frac{a_w {Q_w}^2 + b_w Q_w + c_w}{a_o {Q_O}^2 + b_o Q_O + c_0}$ in classes $D^{\pm}$, $D^{0\pm}$, $D^{\pm\ 0}$

	Class	An example of an equation	General solutions $\frac{dQ_w}{dQ_o} = \frac{a_W {Q_W}^2 + b_W Q_W + c_W}{a_o {Q_O}^2 + b_o Q_O + c_0}$, $Q_O^0 = \frac{b_O}{2a_O}$, $Q_W^0 = \frac{b_W}{2a_W}$
1	D^{++}	$\frac{dQ_w}{dQ_o} = \frac{Q_W(Q_W - 1)}{Q_O(Q_O - 1)}$	$Q_W(Q_O;C) = Q_W^1 - \frac{\sqrt{D_W}}{a_W\left(-1 + C\left\lvert\frac{Q_O - Q_O^2}{Q_O - Q_O^1}\right\rvert^{\rho}\right)}$, $\rho = \sqrt{\frac{D_W}{D_O}} > 0.$
2	D^{+0}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2}{Q_O(Q_O - 1)}$	$Q_W(Q_O;C) = Q_W^0 + \frac{1}{\frac{a_W}{\sqrt{D_O}}\ln\left\lvert\frac{Q_O - Q_O^2}{Q_O - Q_O^1}\right\rvert + C}$,
3	D^{0+}	$\frac{dQ_w}{dQ_O} = \frac{Q_W(Q_W - 1)}{{Q_O}^2}$	$Q_W(Q_O;C) = Q_W^1 + \frac{\sqrt{D_W}}{a_W}\frac{1}{1 + Ce^{\frac{\sqrt{D_W}}{a_O(Q_O + Q_O^0)}}}$
4	D^{00}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2}{{Q_O}^2}$	$Q_W(Q_O;C) = Q_W^0 - \frac{1}{\frac{a_w}{a_o(Q_O^0 - Q_O)} + C}$
5	D^{+-}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2 + 1}{Q_O(Q_O - 1)}$	$Q_W(Q_O;C) = -Q_W^0 + P\tan\left(\frac{\rho}{2}\ln\left\lvert\frac{Q_O - Q_O^1}{Q_O - Q_O^2}\right\rvert + C\right)$, $\rho = \sqrt{-\frac{D_w}{D_o}} > 0, P = \frac{\sqrt{-D_w}}{2a_w}$
6	D^{-+}	$\frac{dQ_w}{dQ_O} = \frac{Q_W(Q_W - 1)}{{Q_O}^2 + 1}$	$Q_W(Q_O;C) = Q_W^1 - \frac{\sqrt{D_W}}{P}\frac{1}{-1 + Ce^{-2\rho\tan^{-1}\left(\frac{Q_O^0 + Q_O}{P}\right)}}$, $\rho = \sqrt{-\frac{D_W}{D_o}} > 0, P = \frac{\sqrt{-D_o}}{2a_o}$
7	D^{--}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2 + 1}{{Q_O}^2 + 1}$	$Q_W(Q_O;C) = -Q_W^0 + P_2\tan\left(\rho\tan^{-1}\left(\frac{Q_O^0 + Q_O}{P_1}\right) + C\right)$, $\rho = \sqrt{\frac{D_W}{D_o}} > 0,\ P_1 = \frac{\sqrt{-D_o}}{2a_o},\ P_2 = \frac{\sqrt{-D_W}}{2a_W}$
8	D^{0-}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2 + 1}{{Q_O}^2}$	$Q_W(Q_O;C) = Q_W^0 + P\tan\left(\frac{\sqrt{-D_W}}{2}\left(\frac{1}{a_o(Q_O^0 - Q_O)} + C\right)\right), P = \frac{\sqrt{-D_W}}{2a_W}$
9	D^{-0}	$\frac{dQ_w}{dQ_O} = \frac{{Q_w}^2}{{Q_O}^2 + 1}$	$Q_W(Q_O;C) = Q_W^0 - \frac{1}{\frac{2a_w}{\sqrt{-D_o}}\tan^{-1}\left(\frac{Q_O^0 + Q_O}{P}\right) + C}, P = \frac{\sqrt{-D_o}}{2a_o}$

Decisive rules were formulated for the first time on the basis of the proposed criteria, allowing timely detection and prevention of the consequences of loss of stability of the displacement front and targeted regulation of the flooding system by stopping, forcing, limiting operating modes, assigning well interventions to producing and injection wells [4-5]. The set of catastrophes of the growth equation is determined by the set of solutions obtained when the discriminant for oil and water are equal to zero. The resulting conclusion for the problem of hydrodynamic effect will take the form of following:

$$D_O = {b_O}^2 - 4a_O c_O = 0, \quad D_W = {b_W}^2 - 4a_W c_W = 0$$

where the coefficients $a_O,\ b_O,\ c_O,\ a_W,\ b_W,\ c_W$ for each oil or water phase are determined by the well-known least squares method (LSM), and the resulting system of algebraic equations is solved by the Gauss method. The discriminant criterion and corresponding well mode selection strategies can be formulated as follows:

- when $D_O < 0$ and $D_W > 0$ (the family of solutions D^{-+} from Table 1), the selection of oil has a trend to increase, and water to decrease, it is recommended to increase the selection of liquid from the well, after studying the potential of pumping equipment;
- when $D_O > 0$ and $D_W < 0$ (the family of solutions D^{+-} from Table 1), the extraction of oil has a downward trend, and water to increase, while a water breakthrough is possible;

- when $D_O < 0$ and $D_W < 0$ (the family of solutions D^{--} from Table 1), the oil and water withdrawals have an upward trend;
- when $D_O > 0$ and $D_W > 0$ (the D^{++} family of solutions in Table 1), oil and water withdrawals have a downward trend.

The proposed methodology is sufficiently mobile and accurate for monitoring and controlling both the state of the flooding process and the well stock, and in general for the development of deposits in the "on-line" mode. The system optimization of flooding of oil fields is designed for a certain technological and economic effect of resource conservation and energy efficiency.

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Stability of a rectangular plate compressed in one direction in the case of inhomogeneous subcritical state with regard to heteromodularity

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The problem of elasticoâ€“plastic stability of plates under inhomogeneous subcritical state essentially is complicated especially in the cases when the properties of the material differently resist to tension and compression.

We will assume that the properties of the material are subjected to the law [1]

$$\varepsilon_{ij} - \delta_{ij}\varepsilon(\sigma,\sigma_u)(\sigma_{ij} - \delta_{ij}\sigma) \quad for \quad \varepsilon_u > \varepsilon_s$$

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad (i,j = 1,2,3)$$

and the functions $\varepsilon(\sigma,\sigma_u)$ and $f(\sigma,\sigma_u)$ in elastic deformation domains take the form [2]

$$\begin{matrix} f(\sigma,\sigma_u) = \frac{1}{2G} \\ \varepsilon(\sigma,\sigma_u) = \frac{\sigma}{3K} \end{matrix} \quad for \ \varepsilon_u \leq \varepsilon_s$$

Here ε_s are the values of intensity of deformations corresponding to the yield points, while

$$G = \frac{G^+ + G^- - (G^- - G^+)sign\sigma}{2}, \quad K = \frac{K^+ + K^- - (K^- - K^+)sign\sigma}{2}$$

The quantities G^+, G^-, K^+, K^- are shear and volumetric extension module, respectively under uniaxial extension and compression.

Let an elastico-plastic plate differently resisting to extension and compression be under the action of a load that causes extension and compression it with the bend in the plane $x_1 0 x_2$. Denote by a, b, h the length, width and thickness of the plate.

In the subcritical state, the stress-strain state is determined by the stress components [3]

$$\begin{aligned} \sigma_{11}^+ &= E^+ A_0 \left(1 - \alpha_0 \frac{x_2}{b}\right) \left[1 - \omega^+(\varepsilon_n)\right] \quad for \quad \sigma > 0 \\ \sigma_{11}^- &= E^- A_0 \left(1 - \alpha_0 \frac{x_2}{b}\right) \left[1 - \omega^-(\varepsilon_n)\right] \quad for \quad \sigma < 0 \\ \sigma_{22} &= \sigma_{33} = \sigma_{12} = \sigma_{31} = \sigma_{32} = 0 \end{aligned}$$

and strain components

$$\varepsilon_{11} = A_0\left(1 - \alpha_0 \frac{x_2}{b}\right), \varepsilon_{22} = \varepsilon_{33} = -\frac{1}{2}\varepsilon_{11},\ \varepsilon_{12} = \varepsilon_{31} = \varepsilon_{23} = 0$$

Here A_0 and α_0 are constant values

$$\omega^{+} = \lambda^{+}\left[1 - \frac{\varepsilon_s^{+}}{A_0\left(1 - \alpha_0 \frac{x_2}{b}\right)}\right],$$

$$\omega^{-} = \lambda^{-}\left[1 + \frac{\varepsilon_s^{-}}{A_0\left(1 - \alpha_0 \frac{x_2}{b}\right)}\right].$$

The equilibrium equations and strain compatibility conditions are satisfied, the constants A_0 and α_0 are determined from the condition of equivalence of external and internal forces.

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Fixed-point iteration method for solution first order differential equations with nonlocal boundary conditions

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One of the most famous iterative methods to solve nonlinear problems is the quasilinearization technique [1]. We consider the system of n-nonlinear coupled differential equations

$$\dot{x}(t) = f(t, x(t)), t \in [0, T] \tag{1}$$

along with boundary conditions

$$Ax(0) + Bx(T) = C. \tag{2}$$

Where A, $B \in R^{n\times n}$ constant matrices with $\det N \neq 0$, $N = A + B$; $f : [0, T] \times R^n \to R^n$ is some given continuous function. The aim of this thesis is to show that the sequence of functions x_n, which are solutions of

$$\dot{x}_{n+1}(t) = f(t, x_n(t)), \tag{3}$$

subject to the boundary conditions

$$Ax_n(0) + Bx_n(T) = C \tag{4}$$

converges to the solution of problem (1)-(3)

Theorem 1. *Let x and x_n, respectively, be the solutions of (1)-(2) and (3)-(4). Assume that f is a nonlinear analytic function. Then, if $MKT < 1$, the sequence of functions x_n converges to the exact solution x in the L_2norm, where*

$$M = \max |f_x(t, x)|, \ K = \max \|G(t, s)\|,$$

$$G(t, s) = sign(t - s) \begin{cases} N^{-1}A, \ 0 \leq s < t, \\ -N^{-1}B, \ t \leq s \leq T. \end{cases}$$

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On one estimate of the maximum modules of a power series in a circle

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We consider a power series $f(z) = \sum a_n z^n$, whose convergence radius equals 1. Let

$$M(r) = \max_{|z|\leq r} |f(z)|, \ \ \mu(r) = \max_{|n} |a_n| r^n, \ 0 \leq r \leq 1.$$

In the present paper we construct a class of functions $\psi(y) > 0$, $y > 0$, for which Wiman-Valiron type estimations are set up:

$$M(r) \leq \psi(\mu(r)).$$

The estimations are fulfilled to within the set of finite lograrithmic measure. In particular, the estimations of the form

$$M(r) \leq \frac{\mu(r)}{c_{(1-r)}\gamma} \left[\log \frac{\mu(r)}{c_{(1-r)}\gamma}\right]^{\frac{1}{2}+\varepsilon}, \ \varepsilon > 0, \ \ \gamma > 0.$$

are valid, the number γ is related to the class of functions $\psi(y)$.

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On Martsinkevich space

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The Martsinkevich space $X = SM_{p,\lambda}(\Omega)$ is considered [1]. It is the Banach space of Lebesgue measurable functions on Ω with the norm ($1 \leq p < +\infty$, $0 \leq \lambda < 1$)

$$\|f\|_{p,\lambda} = \sup_{E \subset \Omega} (\frac{1}{|E|^{\lambda}} \int_E |f|^p \, dt)^{\frac{1}{p}},$$

where sup is taken over all $E \subset \Omega$. In the classical case of Morrey $L_{p,\lambda}(\Omega)$ space, sup is taken over all sets $E = B \bigcap \Omega$, where $B \subset R^n -$is an arbitrary ball. Therefore, is quite obvious that the continuous embedding $SM_{p,\lambda}(\Omega) \subset L_{p,\lambda}(\Omega)$. It is shown that for $0 < \lambda < 1,\ SM_{p,\lambda}(\Omega)$ is an non-separable rearrangement invariant space [2].

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Some exact solutions to problems of nonlinear viscoelasticity

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We cite theorems that by fulfilling the condition of mechanical incompressibility of a material reduce the problem of nonlinear theory of viscoelasticity determining equations to the problem of physically nonlinear theory of elasticity of rheonomical bodies.

Give the statement of a quasistatic problem of nonlinear theory of viscoelasticity determining equations for mechanically incompressible bodies

$$2G_0 e_{ij} = f(\sigma_+) s_{ij} + \int_0^t \Gamma(t-\tau)\ f(\sigma_+) s_{ij} d\tau;\ \theta = 0; \tag{1}$$

or

$$s_{ij} / 2G_0 = \varphi(\varepsilon_+) e_{ij} - \int_0^t L(t-\tau)\ \varphi(\varepsilon_+) e_{ij} d\tau;\ \theta = 0; \tag{2}$$

and

$$\sigma_{ij,j} + F_i = 0;\ \ \sigma_{ij} l_j|\ s_\sigma = R_i;\ u_i|\ s_u = u_{oi}; \tag{3}$$

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2 \ \text{ or } \ \varepsilon_{ij,kl} + \varepsilon_{kl,ij} = \varepsilon_{ik,jl} + \varepsilon_{jl,ik}. \tag{4}$$

Here $i,\ j,\ k,\ l = 1,\ 2,\ 3$; u_i, ε_{ij}, σ_{ij} are the components of permutations, deformation and stress, respectively; $e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}$; $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$; $\varepsilon = \varepsilon_{ij}\delta_{ij}/3$; $\sigma = \sigma_{ij}\delta_{ij}/3$; δ_{ij} are Kronecker symbols: $\varepsilon_+ = (2e_{ij}e_{ij}/3)^{1/2}$; $\sigma_+ = (3s_{ij}s_{ij}/2)^{1/2}$; $G_0 = const$ is an instantaneous shear modulus of a material; f, φ are the functions of nonlinearity of a material; $\Gamma(t)$ and $L(t)$ are mutually-resolvent kernels; F_i and R_i are volumetric and surface forces, respectively; u_{0i} are boundary permutations.

The following theorems are holds.

Theorem 1. Problem (1),(3),(4), has the following solution.

$$u_i = u_i' + \int_0^t \Gamma(t-\tau)\ u_i'\, d\tau;\ \varepsilon_{ij} = \varepsilon_i' + \int_0^t \Gamma(t-\tau)\ \varepsilon_i'\, d\tau,\ \sigma_{ij} = \sigma_{ij}', \tag{5}$$

where the quantities u_i', ε_i', σ_{ij}' are the solutions of the following problem of the theory of nonlinear elasticity of mechanically incompressible bodies

$$2G_0 e_{ij}' = f(\sigma_+')\, s_{ij}';\ \theta = 0; \tag{6}$$

$$\sigma_{ij,j}' + F_i = 0;\ \ \sigma_{ij}' l_j\big|\ s_\sigma = R_i;\ u_i^l\big|\ s_u = u_{oi} = u_{oi}' - \int_0^t L(t-\tau)\ u_{oi} d\tau; \tag{7}$$

$$\varepsilon'_{ij} = \left(u'_{i,j} + u'_{j,i}\right)/2 \;\; \varepsilon'_{ij,kl} + \varepsilon'_{kl,ij} = \varepsilon'_{ik,jl} + \varepsilon'_{jl,ik}. \tag{8}$$

Here we adopt the following denotation

$$e'_{ij} = \varepsilon'_{ij} - \varepsilon'\delta_{ij};\; s'_{ij} = \sigma'_{ij} - \sigma'\delta_{ij};\;\; \varepsilon' = \varepsilon'_{ij}\delta_{ij}/3\,;\, \sigma' = \sigma'_{ij}\delta_{ij}/3\,;\; \theta' = 3\varepsilon';$$

$$\varepsilon'_{+} = \left(2e'_{ij}e'_{ij}/3\right)^{1/2};\quad \sigma'_{+} = \left(3s'_{ij}s'_{ij}/2\right)^{1/2}.$$

Theorem 2. Problem (2),(3),(4) has the following solution

$$u_i = u''_i;\, \varepsilon_{ij} = \varepsilon''_{ij};\;\; \sigma_{ij} = \sigma''_{ij} - \int_0^t L\,(t-\tau)\;\sigma''_{ij}d\tau, \tag{9}$$

where the quantities u''_i, ε''_i, σ''_{ij} are the solutions of the following problem of the theory of nonlinear elasticity of mechanically incompressible bodies

$$s''_{ij}/2G_0 = \varphi\left(\varepsilon''_{+}\right)\; e''_{ij};\theta'' = 0; \tag{10}$$

$$\sigma''_{ij,j} + F_i + \int_0^t \Gamma\,(t-\tau)\;F_i\,d\tau = 0; \tag{11}$$

$$\sigma''_{ij}l_j\big|\; s_\sigma = R_i + \int_0^t \Gamma\,(t-\tau)\;R_i\,d\tau;\quad\; u''_i\big|\; s_u = u_{oi}; \tag{12}$$

$$\varepsilon''_{ij} = \left(u''_{i,j} + u''_{j,i}\right)/2 \qquad \varepsilon''_{ij,kl} + \varepsilon''_{kl,ij} = \varepsilon''_{ik,jl} + \varepsilon''_{jl,ik}. \tag{13}$$

Here we denote

$$e''_{ij} = \varepsilon''_{ij} - \varepsilon''\delta_{ij};\; s''_{ij} = \sigma''_{ij} - \sigma''\delta_{ij};\;\; \varepsilon'' = \varepsilon''_{ij}\delta_{ij}/3\,;\, \sigma'' = \sigma''_{ij}\delta_{ij}/3\,;\; \theta'' = 3\varepsilon'';$$

$$\varepsilon''_{+} = \left(2e''_{ij}e''_{ij}/3\right)^{1/2};\quad \sigma''_{+} = \left(3s''_{ij}s''_{ij}/2\right)^{1/2}.$$

The proof of theorems 1 and 2 are carried out by direct substitution of formulae (5) and (9) into appropriate relations.

Theorems 1 and theorems 2 are also valid for $f = 1$, $\varphi = 1$ that holds in the case of the theory of linear viscoelasticity.

On the Cholodowsky type Szasz-Mirakjan-Bernstein operators

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In this study, we have defined two different Stancu type generalization of Chlodowsky type Szasz-Mirakjan-Bernstein operators and investigated their approximation properties. We have given general moment formulas of these operators and obtained the rate of convergence in terms of modulus of continuity. Also, Voronovskaja type theorems have been obtained.

Dzhabarzade class of Hermition operators

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Our goal is to draw attention to a class of operators that is interesting in our opinion, in Banach space X introduced by R.M.Dzhabarzadeh [1]. This is the only publication in this class since 1977.

We know three earlier generalizations of Hermitian operators from Hilbert space to Banach space:

1) a class of spectral operators 1954. Dunford [2. p.257],

2)the class of $HL(X)$ Hermitian operators, G.Lumer [2, p.95],

3)the class $SA(X)$ of conjugated Abelian operators Stapfli J.

Below we will add some results to the paper [1] that give comparison with Hermitian operators in Hilbert space. We call the linear bounded operator T in Banach space X, RD Hermitian (the clas $RD(X)$, where RD is abbreviation of Rakhshanda Dzhabbarzadeh), if for any $x, y \in X$ and any real number t the following equality is fulfilled:

$$||Tx + ty||^2 + ||x - tTy||^2 = ||Tx - ty||^2 + ||x + tTy||^2 \tag{1}$$

Our first two theorems concern the spectrum of the RD-operator.

Theorem 1. *For $T \in RD(X)$ its spectrum $\sigma(T)$ is real.*

We give two proofs. The first proof uses Berberian's extension of Banach space and reality of a point spectrum (see [1, theorem 2]). It is very short.

The second proof is direct and longer. The used notions in the following theorem (see [4]).

Theorem 2. *The operator $T \in RD(X)$ satisfies the Weyl theorem for the essential Weyl and Browder spectra coinciding between themselves.*

The motive of the following result is Schatten's remark [3, p.18] on Hermitian operators in Hilbert spaces.

Theorem 3. *If in reflexive Banach spaces the operator $T \in RD(X)$ has for some natural n a compact degree T^r, then the operator T itself is compact.*

For Hermitian operator in Hilbert space, Lenduev B. and Stone M.H. indicated the existence of invariant and also netrainvariant subspace. This is suitable for $RD(X)$ operators as well.

Theorem 4. *The operator $T \in RD(X)$ has an invariant and even an ultrainvariant subspace. Hence it follows that if the operator S is quasisimilar to the RD Hermitian operator, then S has an invariant subspace. On notions used in theorem 4 see (15, p. 85 and p.91).*

Theorem 5. *A class of RD Hermitian by Dzhabarzadlh and a class of Lumer'sHα Hermitian generally speaking, are independent of each other, but they can intersect.*

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Normally-unitary operators

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Considering one of the characterizations of a unitary operator in Hilbert space [see[1], p.60], we introduce a new class in Hilbert space. For the used notion see in [2-5]. We call a bounded linear operator T in an orbitrary Banach space X a normally-unitary if T is normally continuously inverti ble and its spectrum lies on a unit circle. We denote this class of operators by $NU(X)$. Enumerate the basic results. The first theorem indicates the place of $NU(X)$ in the hierarchy.

Theorem 1. *$NU(X)$ is properly contained in the class of norm unitary operators, i.e. $T \in U(X)$ is continuously invertible, and $||T|| = ||T^{-1}|| = 1$.*

We give an example when $T \in U(X)$ but $T \notin NU(X)$. The following two theorems describe the structure of the spectrum.

Theorem 2. *Thin structure of the operator $T \in NU(X)$ is described (see: [5]).*

Theorem 3. *The diagram of the Taylor-Halverg states of the operators $T \in NU(X)$ is given. The following two theorems concern essential spectra of the operator $T \in NU(X)$.*

Theorem 4. *Characterization of the essential Coldberg spectrum σ_j; and description of the spectrum of the operator $T \in NU(X)$ is given by $\sigma_j(T)$ the set of normally eigenvalues $\pi_{0\nu}(T)$.*

Theorem 5. *The Weyl theorem for the essential spectra of the operators $T \in NU(X)$ is deduced from the previous theorem.*

The results of [6] on topological closeness of the Bower number image of unitary operator in Hilbert space are transferred on two classes of Banach spaces.

Note that the result of [6] was obtained by means of the spectral theorem, in particular the results of [6] obtain a new proof.

Theorem 6. *For the operator $T \in NU(X)$ in a reflexive smooth or in reflexive round Banach space X, the Bauer number image $V(T)$ is topologically closed if and only if the spectrum T coincides with the set of its eigenvalues.*

In [7] a class of parenthesis operators in Hilbert space is introduced. Following it, we introduce them in Banach space. We call the operator $T \in NU(X)$ a parenthesis operator of its spectrum lies in an open arch of a unit circle with an angle less than π.

The following simple geometrical fact is used when studying operators.

Theorem 7. *Let $C \in NU(X)$, then C will be parenthetic if and only if the closure of the Bauer number image $V(T)$ does not contain a zero.*

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Assessment of informativity data of the Water Flow in observation points by Schulz method

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Using the method of Schulz, assess informativeness of water consumption data at specific hydrometric observation posts in Lankaran rivers of the natural area. Assessment of informativeness of data on water consumption of stations is possible on the basis of the following methodology:

1. To assess the informativeness of the data on water consumption obtained from each point, first of all, a hydrological series (hydrological measurement data) describing the change in water consumption (average monthly) over time in different years in hydrological points is used;

2. An appropriate entropy value is calculated for each site for each year, which characterizes the uncertainty of the hydrological sequence;

3. According to the formula $I(A, B) = H(B) - H(A)$ described above, an information matrix characterizing the variability of water consumption in the settlements is constructed. $A(H)$ and $B(H)$ used in formula $I(A, B) = H(B) - H(A)$ are the entropy values of the states of the hydrological series for each year and the following year at the points, respectively.

4. Using the information matrix, a matrix of relations between the points is constructed;

5. Decisions are made to assess informatization between polling stations using collective decision-making methods.

The Schulz method is used as a method of collective decision-making [1].

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Lamb waves in a three-layer slab of compressible material

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Among the problems related to nonlinear dynamic effects, a significant place is occupied by problems related to elastic bodies with initial stresses (strains). It should be noted that initial deformations occur in structural elements as a result of technological operations during their manufacture and assembly, in composite materials during technological processes of their creation and etc. In this paper is considered the problem of propagation Lamb waves in a three-layer slab of compressible material with finite initial strains, using the three-dimensional linearized theory of elastic wave propagation in bodies with initial stresses. We consider a three-layer slab and assume that the thicknesses and materials of the front layers of the slab are the same. We accept that the ratio of the elasticity of the material of the layers is determined through the potential of the harmonic type

$$\Phi^{(r_n)} = \frac{1}{2}\lambda^{(r_n)}\left({S_1}^{(r_n)}\right)^2 + \mu^{(r_n)}{S_2}^{(r_n)}, \tag{1}$$

where

$${S_1}^{(r_n)} = \sqrt{1+2{\varepsilon_1}^{(r_n)}} + \sqrt{1+2{\varepsilon_2}^{(r_n)}} + \sqrt{1+2{\varepsilon_3}^{(r_n)}} - 3$$

$$\begin{aligned} {S_2}^{(r_n)} = \left(\sqrt{1+2{\varepsilon_1}^{(r_n)}}-1\right)^2 + \left(\sqrt{1+2{\varepsilon_2}^{(r_n)}}-1\right)^2 + \\ + \left(\sqrt{1+2{\varepsilon_3}^{(r_n)}}-1\right)^2 \end{aligned} \tag{2}$$

In (1) and (2) $\lambda^{(r_n)}$, $\mu^{(r_n)}$are the material constants $r_n$layer of the plate, ${\varepsilon_i}^{(r_n)}$ $(i=\overline{1,3})$ are the main values of the tensor Green strain.

Within the framework of the above assumptions, is investigated the propagation of Lamb waves in a three-layer slab. The investigation is carried out using the coordinate system associated with the initial state of the body. Similarly [1,2] the obtained differential equations are solved. An algorithm is developed for the numerical solution of the dispersion equation, which is obtained from the condition for the existence of nontrivial solutions. In specific cases, graphs will be constructed for the mechanical and geometric parameters of the slab

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Reducing a boundary value problem for a linear, loaded first-order linear loaded differential equation with constant coefficients to the Cauchy problem

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Let us consider the following boundary problem:

$$y'(x) + ay(x) = f(x) + by(1), \in (0,1) \tag{1}$$

$$\alpha y(0) + \beta y(1) = \gamma, \tag{2}$$

where a, b, α, β and γ are the given constant numbers, $f(x)$, are the given continuous It is easy to see that

$$Y(x-\xi) = e^{xa(x-\xi)}\theta(x-\xi), \tag{3}$$

is a fundamental solution of the equation

$$y'(x) - ay(x) = F(x), \tag{4}$$

where $\theta(t)$ is a Heaviside unit function [2]. Indeed

$$Y'(x-\xi) - aY(x-\xi) = \delta(x-\xi), \tag{5}$$

where $\delta(t)$ Dirac delta function. Multiplying equations (1) by the fundamental solution (3), integrating with respect to and applying the formula of integration by parts we obtain [1]:

$$\int_0^1 y'(x)Y(x-\xi)dx + a\int_0^1 y(x)Y(x-\xi)dx =$$

$$= \int_0^1 f(x)Y(x-\xi)dx + by(1)\int_0^1 Y(x-\xi)dx,$$

or

$$y(1)e^{a(1-\xi)}\theta(1-\xi) - y(0)e^{-a\xi}\theta(-\xi) - \int_0^1 y(x)Y'(x-\xi)dx +$$

$$+a\int_0^1 y(x)Y(x-\xi)dx = \int_0^1 f(x)Y(x-\xi)dx + by(1)\int_0^1 Y(x-\xi)dx$$

$$y(1)e^{a(1-\xi)}\theta(1-\xi) - y(0)e^{-a\xi}\theta(-\xi) - \int_0^1 f(x)e^{a(x-\xi)}\theta(x-\xi)dx -$$

$$-by(1)\int_0^1 e^{a(x-\xi)}\theta(x-\xi)dx = \int_0^1 y(x)\left[Y'(x-\xi) - aY(x-\xi)\right]dx =$$

$$= \int_0^1 y(x)\delta(x-\xi)dx = \begin{cases} y(\xi), \xi \in (0,1), \\ \frac{1}{2}y(\xi), \xi = 0, \xi = 1. \end{cases} \tag{6}$$

From the main relation (6), we have:

$$\frac{1}{2}y(0) = y(1)e^a - \frac{1}{2}y(0) - \int_0^1 f(x)e^{ax}dx - by(1)\int_0^1 e^{ax}dx, \tag{7}$$

$$y(\xi) = y(1)e^{a(1-\xi)} - \int_\xi^1 f(x)e^{a(x-\xi)}dx - by(1)\int_\xi^1 e^{a(x-\xi)}dx, \xi \in (0,1). \tag{8}$$

Considering (2) and (7) as system of algebraic equations, we obtain:

$$\begin{cases} \alpha y(0) + \beta y(1) = \gamma \\ y(0) + y(1)\left[-e^a + b\frac{e^a-1}{a}\right] = -\int_0^1 f(x)e^{ax}dx \end{cases} \tag{9}$$

$$\Delta = \begin{vmatrix} \alpha & \beta \\ 1 & \frac{(b-a)e^a - b}{a} \end{vmatrix} = \frac{\alpha(b-a)e^a - \alpha b}{a} - \beta \neq 0, \tag{10}$$

Then the stated boundary value problem (1)-(2) is reduced to the following Cauchy problem:

$$y'(x) + ay(x) = f(x) - \frac{b}{\Delta}\begin{vmatrix} \alpha & \gamma \\ 1 & -\int_0^1 f(x)e^{ax}dx \end{vmatrix} > 0, \tag{11}$$

$$y(0) = \frac{1}{\Delta}\begin{vmatrix} \gamma & \beta \\ \int_0^1 f(x)e^{ax}dx & \frac{(b-a)e^a - b}{a} \end{vmatrix}, \tag{12}$$

Theorem. *If a, b, α, β and γ are constant numbers and a $f(x)$ is a given constant function, then under condition (10) the boundary value problem (1)-(2) is reduced to the Cauchy problem (11)-(12).*

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